Dynamic Equivalencing of Distribution Network with Embedded Generation

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Abstract

Renewable energy generation will play an important role in solving the climate change problem. With renewable electricity generation increasing, there will be some significant changes in electric power systems, notably through smaller generators embedded in the distribution network.

Historically insignificant volumes of Embedded Generation (EG) mean that traditionally it has been treated by the transmission system operator as negative load, with its impact on the dynamic behaviour of power systems neglected. However, with the penetration level increasing, EG would start to influence the dynamics and stability of the transmission network. Hence the dynamic behaviour of distribution network cannot be neglected any more.

In most cases, a detailed distribution network model is not always available or necessary for the study of transmission network dynamics and stability. Thus a dynamic equivalent model of the distribution network that keeps its essential dynamic behavior, is required.

Most existing dynamic equivalencing methods are based on the assumption that the detailed information of the complete power system is known. Dynamic equivalencing methods based on coherency of the machines have been applied to transmission networks but cannot be applied to distribution networks due to their radial structure. Hence an alternative methodology has been developed in this project to derive the dynamic equivalent model of the distribution network using system identification, without the detailed information of the distribution network necessarily known.

Case studies have been accomplished in PSS/E on a model of the Scottish transmission network with the distribution network in Dumfries and Galloway. Embedded generation with a certain penetration level in either conventional generation or DFIG wind generation has been added to the model of the distribution network. The dynamic equivalent models of the distribution network are compared with the original distribution network model using a series of indicators. A constant power model has also been involved in the comparison to illustrate the advantage of using the dynamic equivalent to represent the distribution network.

The results suggest that a proper dynamic equivalent model derived using this methodology may have better agreement to the original power system dynamic response than constant power equivalent. A discussion on factors

that influence the performance of the dynamic equivalent model, is given to indicate the proper way to use this methodology.

The major advantage of the dynamic equivalencing methodology developed in this project is that it can potentially use the time series obtained from measurements to derive the dynamic equivalent models without knowing detailed information on the distribution network. The derived dynamic equivalent, in a simple spate-space form, can be implemented in commercial simulation tools, such as PSS/E.

Declaration of Authorship

I, Xiaodan Feng, declare that the work presented in this thesis is my own, and generated by me as the result of my own original research. I confirm that:

- this work was done wholly or mainly while in candidature for a research degree at this university;
- where any part of this thesis has previously been submitted for a degree or any other qualifications at this university or any other institution, this has been clearly stated;
- where I have consulted the published work of others, this is always clearly attributed;
- where I have quoted from the work of others, the source is always given. With the exception of such quotations, this thesis is entirely my work;
- I have acknowledged all main sources of help;
- where the thesis is based on work done by myself jointly with others, I have made clear exactly what was done by others and what I have contributed myself.

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Xiaodan Feng

Dedication

I would like to dedicate this work to my beloved parents and my Buddhist guru Khejok Rinpoche.

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Abbreviations

AC	Alternating Current
ANN	Artificial Neural Network
ARMA	Auto-Regressive Moving Average
ARIMAX	Auto-Regressive Moving Average with Integrator in the noise model and eXogenous
ARX	Auto-Regressive eXogenous
CCT	Critical Clearing Time
CHP	Combined Heat and Power
DC	Direct Current
DFIG	Doubly-Fed Induction Generator
D&G	Dumfries and Galloway
DYNRED	DYNamic REDuction
EG	Embedded Generation
EPRI	Electric Power Research Institute
ERA	Eigensystem Realization Algorithm
ESA	Evolutionary Strategy Algorithms
EU	European Union
FDM	Frequency Dependent Model
GA	Genetic Algorithms
GW	GigaWatt
IGBT	Insulated Gate Bipolar Transistor
IME	Interarea Model Estimation
IOM	Input/Output Model
MATLAB	MATrix LABoratory
MIMO	Multi-Input-Multi-Output
MSE	Mean Square Error
MW	MegaWatt
N4SID	Numerical algorithms for Subspace state-space system IDentification
NGC	National Grid Company
PBM	Physics Based Model
PEM	Prediction Error Minimisation
PF	Power Factor
PMUs	Phasor Measurement Units
PSS/E	Power System Simulator for Engineering
RO	Renewables Obligation

SME	Synchronic Modal Equivalencing
SP	Scottish Power UK Plc.
SP	Structure Preserving
SPM	Series Parallel Model
SSE	Scottish and Southern Energy Plc.
SVD	Singular Value Decomposition
UK	United Kingdom
VDM	Voltage Dependent Model

Symbols

$e(kT)$	measurement noise
f	system frequency
g	generator buses that are retained
H_{eq}	equivalent generator's inertia
I	current
$\bar{I}$	bus current
I_T	node injection current at the connecting bus
$\tilde{I}_i$	phasor current flowing into the network at node i
$Ke(kT)$	process noise
l	load buses to be eliminated
P	real power
P_e	generator electric power
P_{EG}	generation associated with the distribution feeder
P_{load}	distribution feeder load
Q	reactive power
S	apparent power
S_i	power injection at node i
T	matrix containing eigenvectors
$u(kT)$	input at time instant kT
$\bar{U}$	bus voltage
U_T	node terminal bus voltage at the connecting bus
V	voltage
V_a	voltage at fictitious node a
V_i	voltage at node i
$\tilde{V}_i$	is the phasor voltage to ground at node i
$\bar{W}$	sensitivity matrix
$x(kT)$	current state at time instant kT
X	original state variables
$\bar{I}_g^{eq}$	equivalent current injection vector

$y(kT)$	output at time instant kT
Y	steady state admittance
$\bar{Y}$	bus admittance matrices
Y'	transient admittance
Y''	sub-transient admittance
Y_a	admittance of fictitious node a
$\bar{Y}_{gg}^{eq}$	equivalent bus admittance matrix
Y_i	admittance of node i
Y_{ii}	self admittance of node i
Y_{ij}	mutual admittance between nodes i and j
δ_i	rotor angle of generator i
δ_j	rotor angle of generator j
δ_{ij}	phase angular difference between the two generators
Δ	the small deviation from initial value
$\Delta\delta_g$	machine angle increment
Λ	the diagonal matrix of eigenvalues

Contents Summary

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Chapter 1

Introduction

1.1 Thesis Background

1.1.1 Renewable Energy Development

Renewable energy is derived from natural sources like wind, waves, tides, the sun and biomass, etc. Using renewable sources helps in reducing reliance on fossil fuels and also in cutting down the production of carbon dioxide and other green house gases. It will play an important role in solving the climate change problem which has already raised concern all over the world.

Europe is on a path towards renewable generation. The UK government, for example, is working for the ambitious target to cut its carbon emissions to 80% below 1990 levels by 2050, with an intermediate target of at least 26% by 2020 [1]. To deliver the 2020 target, around 30% of electricity supply is suggested to be generated from renewables [2].

The integration of renewable energy technologies into the network would lead to some significant changes in power systems.

1.1.2 Embedded Generation

The current UK energy system is a highly centralised system, in which most large power stations are connected to the transmission network and then transported to the loads through the distribution networks.

Since most renewable energy sources are geographically widely distributed, renewable generators may locate far from the transmission network connections but close to the loads. Therefore they need to be embedded into the distribution networks as Embedded Generation (EG or distributed generation).

EG can be used to meet the requirement of load growth and to help relieve transmission constraints, whilst offering higher power quality and overall reliability at a competitive cost [3]. It has the potential to be a long term alternative or supplement to the present highly centralised system [4].

On the other hand, distribution networks may face more problems caused by embedding renewable generation. The use of EG may have a significant impact on transmission system stability problems at high penetration levels [5]. It has the potential to cause voltage oscillations and interfere with voltage-control processes [6].

1.2 Project Objectives and Scope

This project focuses on deriving proper dynamic equivalents, which can give reasonable approximations to the dynamic behaviour of a distribution network with embedded generation and also to its impacts on the transmission network. The methodology used to derive the equivalent is based on simulation and potentially measurements at selected points of the power system.

The main advantage of this equivalencing methodology is that the detailed structure of the distribution network and the parameters of the components within it do not necessarily need to be known. This is helpful when some new generating units are introduced into the distribution network with their capacities unknown or technologies unfamiliar to the transmission system operator.

1.2.1 Dynamic Equivalencing of Distribution Network

In most cases, a detailed distribution network model is not necessary for the study on transmission network dynamics and stability problems. The overall combined model of transmission and distribution network would be too complicated or too large to be represented in full.

Traditionally, distribution networks have been equivalenced as loads. However, with more embedded generation implemented into the distribution network, it becomes improper to ignore the impact of embedded generators on the system and to assume the distribution network as just a load.

To simplify a complex power system model, dynamic equivalencing has usually been applied. Dynamic equivalencing aims to eliminate part of the system model and replace it by a dynamic equivalent which has close enough dynamic behaviour to the original part. For the reason mentioned above, the dynamic equivalents of distribution network are needed for dynamic and stability studies. A dynamic equivalencing method which simplifies the distribution network without losing its dynamic features would become useful.

1.2.2 Problem Statement

The trend of electricity generation is from the current power system based on centralised generators to a system that also has a large number of small or middle-sized embedded generators. Due to the increased penetration of embedded generation, it may be inappropriate to represent distribution networks by a simple load model.

With embedded generators integrated into distribution networks, the distribution network cannot be regarded as a ”passive” network (which has no source of energy). The dynamics of embedded generators in a distribution network could interact with the dynamics of machines and control devices in the transmission network. The necessary accuracy could be lost if it was modelled as a passive load [7].

Embedded generators increase the complexity of the distribution network especially because of the complexity and variety of the new technology devices. Problems could be encountered when the new technology devices are implemented using commercial software which have not yet developed the corresponding models (e.g.: models of some renewable generators might not have been validated). This will lead to increasing difficulty in modelling the distribution network in a detailed way for dynamic studies, and would result in computational difficulty in simulations.

Also, the data of the distribution network is not always available. The lack of data on components in the distribution system would limit the applicability of conventional dynamic equivalencing methodologies on distribution networks.

For the reason that the information of the distribution network with embedded generators is not always known in detail, this research project was aimed at developing simulative and ultimately measurement-based equivalencing methods to meet the requirement of the new power system studies.

1.2.3 Project Objectives

This project was undertaken to support the renewable energy policy for the UK, concentrating on dynamic equivalencing of distribution networks with embedded generation. With the increasing modelling difficulty caused by embedded generation, this project would become meaningful for dynamic analysis and stability studies on the new power system. It has several distinct objectives:

- To develop a dynamic model of the Scottish power system, including a detailed model of a distribution network in Dumfries and Galloway. Embedded generators (renewable or non-renewable) are implemented into the distribution network model, with the assurance that the model works stably under normal operating conditions and could satisfy the requirement of many investigations.
- To analyse the dynamics of the distribution network with embedded generators and identify possible stability problems caused by its interaction with the transmission network.
- The ultimate goal of this project is to develop a methodology for deriving proper dynamic equivalent models of distribution networks with embedded generation. These dynamic equivalent models could be used for transmission-level stability studies and the methodology applied to other distribution networks.

1.3 Thesis Contribution to Knowledge

Overall, the project has developed a new dynamic equivalencing methodology for distribution networks, which considers the influence of the embedded generation on the system. This methodology is based on system identification theory, which derives the dynamic equivalents from simulations and measurements.

The principal advantage of this methodology is that detailed data on the distribution system does not necessarily need to be known. It solves the increasing complexity and data availability problems caused by embedding more small generators in the distribution system and could probably also be used for deriving dynamic equivalents of the transmission network.

This methodology has been tested on a Scottish power system model. The derived dynamic equivalents have been verified with acceptable accuracy.

1.4 Thesis Outline

The thesis consists of nine chapters, together with necessary appendices.

Chapter 2 presents an overview of the power system and changes to the current energy scheme. The development of embedded generation and its resultant impact on the grid are of special concern.

Chapter 3 introduces the general stability problems of power systems. The impacts of increased embedded generation on system stability are studied which identifies the importance of considering embedded generation impact in the dynamic equivalencing of a distribution network.

Chapter 4 details the state of art on dynamic equivalencing. The existing approaches are examined along with their limitations.

Chapter 5 specifies system identification theories, its scientific basis, along with the major steps of the methodology used to derive the dynamic equivalent models.

Chapter 6 describes the implementation of a case study power system model in the commercial software package Power System Simulation for Engineering (PSS/E) used for dynamic simulations. Potential approaches are considered with a view to data availability, software complexity and modelling practicality.

Chapter 7 examines the results of the case study in PSS/E. The methodology was applied to a distribution system with embedded conventional generators. The performance of the derived dynamic equivalent models were compared with that of a constant power equivalent model.

Chapter 8 highlights the issues associated with the model accuracy. Application on a distribution network with embedded DFIG wind generators has also been examined, together with suggestions on disturbance and model selections for deriving dynamic equivalents.

Finally, in Chapter 9, conclusions are drawn regarding the future of dynamic equivalencing and its role. Several suggestions are presented for possible future work on this subject.

Chapter 2

Electric Power Systems

2.1 Introduction

The size and composition of electric power systems may be different, but they have some common functions [8]. Firstly, they generate electricity. Mechanical energy is converted into electric energy in power plants where various technologies are used to generate electricity from primary energy sources such as fossil, nuclear, and hydraulic. After being generated, the electric energy is transported to the points of consumption (or loads) through two levels of networks: transmission and distribution. These two networks are operated at different voltages and work cooperatively to deliver the electricity in a safe, reliable and economic way. Besides the functions above, a power system also has other functions like protection and metering.

2.2 Electric Power Transmission and Distribution

For environmental reasons large power stations usually site at places that are distant from load centres. Also, some renewable generation technologies (such as wind, tidal or wave) are unlikely to be located close to load centres. To transmit the bulk of electric power over significant distances from power stations to load centres, the transmission network is used. Transmission networks are operated at high voltages which allow lower losses and lower capital cost per unit transmitted. The transmission of electricity at high voltages is more efficient than that at low voltages. Distribution networks, which are operated at lower voltages, are used to transmit electricity between substations and customers spread over an area.

In the UK, the National Grid Company (NGC) owns the electricity transmission network in England and Wales and operates the entire transmission system throughout Great Britain [9]. Within Scotland there are two vertically integrated energy companies, Scottish Power (SP) and Scottish and Southern Energy (SSE). Different subsidiary companies of the two own the transmission system in their respective areas and operate the distribution systems in Scotland.

2.2.1 Voltage Level

The electric power system in the UK operates at the following nominal voltages:

- Transmission voltages
  EHV: $275kV$, $400kV$
  HV: some $132kV$ (in Scotland)

- Distribution
  HV: $132kV$
  MV: $33kV$, $20kV$ (not widely used), $11kV$
  LV: $400V$
  Some earlier voltages still exist

HV: $66kV$

MV: $25kV$, $22kV$, $6.6kV$, $3kV$

The transmission network in Great Britain is operated at $400kV$ and $275kV$. It is also referred to as the "Supergrid". The level below, $132kV$, is regarded as sub-transmission network. In Scotland all these three voltage levels are considered to form the transmission network.

The distribution network is normally operated at $33kV$, $11kV$ and $400V$ three-phase. The distribution network starts at a step down transformer at a transmission supply point, feeding a number of distribution lines. A series of transformers, which step the voltage further down, are located along the route. Customers are supplied at 400V three-phase or 230V single phase voltage.

2.2.2 Network Structure

Apart from the voltage level, the structure of the transmission network and that of the distribution network are also different. The former can be interconnected but the latter is normally radial.

Structure of Transmission Network

An interconnected transmission network provides the major national electrical links between all the system participants (generators and loads). Linking these participants through an interconnected network makes it possible to select the cheapest available generation. Hence the market participants are able to trade with the most competitive suppliers, which allow the system operator to accept the most attractive bids to meet the electricity demand [9].

Structure of Distribution Network

Distribution networks are typically radial or interconnected [9]:

- A radial network starts from the grid supply point and goes through the network area without any normal connections to other supplies. It is typically in rural lines for supplying isolated load areas. In a radial distribution network each component has a unique path towards the source of supply.
- An interconnected network has multiple connections to the grid supply points. These points of connections are normally open but various configurations are available with the switches being closed or open. The benefit of an interconnected network is that for maintenance, or during a fault, a small area of the network can be isolated and the rest can keep on supplying power.

Most distribution networks in the UK are radial networks, or if interconnected networks are constructed, as mentioned above, they will operate in the same way as radial networks by using normally-open connection points. There are many long $132kV$ and $33kV$ radials in Scotland [9].

2.3 Electric Power Generation

Electricity can be generated by burning fossil fuels or nuclear fission. Such generation is not renewable because the amount of fuel is finite and could be used up. On the contrary, energy sources like hydro, wind, tidal, geothermal, biomass, and solar are infinite and are regarded as renewable forms of energy.

2.3.1 Conventional Generation

Conventional power stations are generating electricity from the widely used primary energy sources, which include fossil fuel, nuclear fission and falling water [10]. Most of the electricity consumed in the UK is generated from large power stations running on coal, natural gas and nuclear power. These large power plants are connected directly to the transmission network.

Part of the conventional generation is gradually being replaced by renewable generators. Compared to renewable generation, conventional generation has some downsides or constraints [11]:

- Firstly, fossil fuel and nuclear power generation has negative impacts on environments, such as global warming and nuclear waste problems.
- Fossil fuel and nuclear power generation are using finite energy sources.
- Energy sources around the world are not distributed evenly. Countries which lack primary energy sources would become highly dependent on importing them from other countries that have surpluses. As a result, when the latter are supporting or struggling against a certain view, they can put pressure on the former.
- Hydro generation does not have the downsides mentioned above but it is difficult to supply the whole electricity demand solely by hydro generation. Potential hydro generation usually locates at distant sites with some difficulty for access and will increase the difficulty in power transmission.
- In addition, constructing dams and basins sometimes forces many residents to move.

2.3.2 Renewable Generation

Renewable generation such as wind, wave, tidal and solar power have infinite primary energy sources and relatively insignificant impact on the environment. Due to these advantages, many countries are promoting renewable generation although it is usually more expensive and less flexible than conventional power generation.

Scotland has a very promising future in renewable generation. It has 25% of Europe's wind resource and even greater proportions of its tidal and wind resources. There is also plenty of solar radiation on the roofs across Scotland most days of the year [12].

In the UK, renewable energy is a strategy of government to solve the climate change problem. The goal for renewable energy is to make an increasing contribution to energy supplies and the government is working towards a target that renewable generation provide 30% of the UK electricity supplies by 2020 [2].

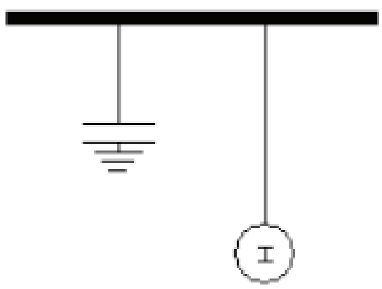
In order to achieve this goal, the government placed a Renewables Obligation (RO) on licensed electricity suppliers to source an increasing proportion of electricity from renewables. Suppliers must meet their obligations by purchasing an amount of electricity from renewable sources, or if not, pay an equivalent amount into a fund. For Great Britain, the proportion in 2010 is 10% of electricity sales and 15% in 2015 [9]. The RO is designed to encourage renewable generation in the electricity market and has provided a significant boost to the economics of renewables.

2.3.3 Wind Generation

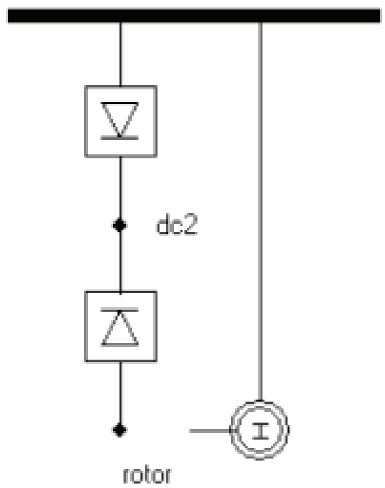
Introduction

Compared to other renewable sources such as wave, tidal and photovoltaics, wind power is relatively cheap. Wind farms may be embedded or directly connected and are classified as large, medium or small power stations. According to National Grid [9], the installed capacity of directly connected wind farms is $2.9GW$ and the installed capacity of embedded medium and small wind farms is $2.1GW$ in 2009/10. The installed capacity of wind farms could reach $16.0GW$ by 2014/15 [9]. Large amounts of wind generation will be connected via the distribution network.

Types of Wind Generator



(a)



(b)

Figure 2.1: Types of Wind Turbines: (a) Fixed-speed generator; (b) Variable-speed generator

Wind turbine generators can be categorised into two main types, fixed-speed and variable-speed.

A fixed-speed wind turbine generator is a typical induction generator, which has high efficiency running at fixed mechanical speed [13][14]. The most common type of variable-speed generation is the Doubly-Fed Induction Generator (DFIG) (Figure 2.1(b)). DFIG consists of a wound rotor induction generator and an AC/DC/AC converter using Insulated Gate Bipolar Transistors (IGBT) to feed power bi-directionally to the rotor circuit [15]. The stator winding is connected directly to the grid while the rotor is fed at variable frequency through the AC/DC/AC converter.

Low power wind turbines are usually built to operate at fixed speed, whilst high power wind turbines are capable of variable speed operation. It has been recognised that many large wind farms are using DFIG variable speed wind turbines and more are under construction. Compared to fixed-speed generators, DFIGs have several advantages [14][16][17]:

- The major advantage of DFIG is its higher efficiency since it has better ability to capture the wind energy by changing the turbine speed. DFIG could optimise the turbine speed while minimizing mechanical stresses on the turbine during gusts of wind. The optimum turbine speed which extracts maximum mechanical energy from the wind varies with the wind speed.

- DFIG may also have better power quality. The power output of the unit is relatively constant as it can store the energy in a gust of wind in the shaft.

- Furthermore, induction machines absorb reactive power. This reactive power is necessary for the equipment to operate correctly but is undesirable. Fixed-speed induction generator require capacitors to provide reactive power support to control and maintain the voltage. DFIG technology with power electronic converters has the ability to regulate the power factor without installing capacitor banks.

- The speed and power factor control within the DFIG system assists in improving turbine and network stability.
- Most of the likely network requirements can be met by variable-speed wind turbine technology, with some insignificant costs in terms of development effort and additional control and communication capability. To meet these requirements, fixed-speed wind turbines are likely to face higher costs than variable-speed wind turbines.

Wind Turbine Modelling

Network studies on wind turbine generators have not been extensive due to their small contributions. Traditionally, they are treated as negative loads [9], which provide nothing but energy to the power system. However, with changes to grid codes, wind generators may start to influence the dynamics of electric power system in the near future, by interacting with conventional generators and loads. Consequently, wind turbine generators should be described properly in dynamic simulations, in response to sudden events (e.g.: transient events), which cannot be adequately simulated beforehand. For this reason, many investigations are ongoing concentrating on building validated dynamic models of wind turbines.

Progress is being made on accurately modelling DFIG, including modelling wind turbines [18][19][13][14]; their associated controller and protection circuits [20][21] and the initialisation methods of wind turbine models [18][22]. Modelling methods of fixed-speed generators are also presented in some papers [23][24]. To avoid the necessity of developing a detailed model of a wind park with hundreds of wind turbines, some results of studies on aggregated models for wind parks can also be found [25].

2.4 Embedded Generation

2.4.1 Introduction

Many of the renewable technologies are considerably smaller-sized compared with conventional power generation. Most renewable energy sources are geographically widely distributed as e.g. wind farms must be located in the windy areas. As a result, many renewable generators could locate distantly from the transmission network connections but close to the loads. These generators, typically with capacity less than $50MW$ are often connected to the medium voltage distribution network rather than the high voltage transmission network. They form embedded generation, or EG.

2.4.2 Types of Embedded Generation

Embedded generation is not centrally planned or centrally dispatched by the utility. It may be large but is more likely to be medium or small (smaller than $50-100MW$). Much of the existing and future EG is either in the form of Combined Heat and Power (CHP) projects or in the form of renewable projects (e.g.: wind).

A CHP plant can simultaneously generate usable heat and electric power in a single process and are generally fuelled by gas, coal or oil. CHP schemes tend to be located close to customers in order to take the heat output.

Renewable generation technologies cover a range of energy sources including hydro, biofuels, wind, wave, tidal and solar. UK figures show that in 2006 biofuels accounted for about half (51%) of renewable energy production, with the other half mainly shared between hydro (25%) and wind (23%). Three years earlier the equivalent percentages were 58%, 30% and 12% [9].

2.4.3 Reasons for Embedded Generation Implementation

The reasons for the development of embedded generation are as follows:

- The implementation of renewable generation has contributed to the development of EG.
- Embedded CHP generators can improve the overall energy efficiency. Transporting heat over a long distance is not economical and consequently it is necessary for a CHP plant to be located close to the heat load. These geographically distributed small generators are hence connected to distribution networks.
- The commercial structure of the electricity supply industry also plays an important role in the EG development. A deregulated environment and open access to the distribution network are likely to provide greater opportunities for EG.
- EG can increase the diversity and the reliability of energy supplies, and potentially lead to more competitive energy markets. It will possibly become a long-term alternative or supplement to the current highly centralised system.
- EG is sited close to the load, which may reduce transmission costs and losses.

2.4.4 Information Availability of Embedded Generation

EG has an overall effect on the Great Britain transmission network and its operation. When planning the development of the transmission system, the output of EG is taken into consideration. However, the outputs of EG power stations cannot directly be seen by the transmission system operator (National Grid) and are outside its control. What the transmission system operator can see is the national system frequency, which shows whether the total demand is met by the total generation supplied to the grid.

Detailed information on EG can be obtained from the relevant distribution network operators but there are no obligations for them to do so. Since the information is voluntarily sourced by the distribution network operators, it could provide a useful initial indicator to the types and capacity of EG connected to distribution networks. However some of it has not been updated adequately and the accuracy of the information cannot be guaranteed by the transmission network operator and hence should not be relied on [9].

2.4.5 Impact of Embedded Generation

The total installed capacity of embedded generation in the UK is $7454MW$ in 2009/10. Of this, some $769MW$ is located in Scotland, $3344MW$ is located in northern zones of England and Wales and $3341MW$ is located in southern zones [9].

The penetration of EG is a parameter for the distribution network. Penetration means the proportion of the distribution feeder load P_{load} being supplied by generation P_{EG} associated with the distribution feeder [26]:

$$\%Penetration = \frac{P_{EG}}{P_{load}} \times 100$$

With penetration of EG increasing, the impacts of EG on power systems cannot be neglected.

Structure of the Power System

Large scale implementation of EG leads to a trend from the current vertically-operated power system (which is supplied by large centralised generators at the transmission network level) towards a future power system comprising a large number of smaller-sized generators embedded into the distribution network.

Power Flow Direction

The current distribution network was designed to accept bulk power from the transmission network and then distribute it to customers. Usually, the power flow from the higher voltage to the lower one. With the penetration of EG increasing, the direction of power flow can become reversed. Hence the distribution network can no longer be regarded as a passive circuit supplying load but an active system with power flows and voltages determined not only by the load but also by EG. Such changes in power flow may have significant impacts on the power system [27]. In addition, EG can unload lines and reduce losses. On the other hand, the reversed power flows from large EG may even raise the losses [11].

Dynamic behaviour

Many of the EG technologies are different from synchronous generator (e.g.: wind turbine generators coupled to the grid through a power electronic converter). When the penetration level of EG is low, the impact of EG on power system dynamics is insignificant. With the penetration level becoming higher, the impact of EG on the dynamics of the power system should not be neglected [27]. Such effects of EG are suggested to be dependent on its technologies [10]. The dynamic issue will be discussed in Chapter 4.

External Impedance

Various topologies of distribution network allows various way of connecting EG to it. This will affect the impedance between the EG and the transmission network, or the connection strength. With a high penetration of EG, the connection strength may influence the transmission network transient stability in a event of fault [10].

2.5 Conclusion

In this chapter, the current trends in power systems changes were reviewed with particular attention to techniques relevant to EG.

Renewable generation has attracted a lot of interest due to environmental considerations and is supported by the UK government. With the rapid development of renewable generation, the present vertically-operated power system is being replaced by power system with a large amount of EG units in distribution networks. Hence the distribution network cannot be regarded as a passive load and starts to influence the dynamic behaviour of the power system.

Chapter 3

Power System Stability

3.1 Introduction

Power system stability is the ability of a power system to maintain a state of operating equilibrium under normal operating conditions and to approach an acceptable state of operating equilibrium after being affected by a disturbance [27].

Traditionally, the stability issue was neglected when assessing embedded generation schemes, since the distribution network is passive and remains stable under most circumstances, provided the transmission network is stable. However, the influence of EG on the power system depends on its penetration. With the trend for renewable generators being widely used, the amount of EG introduced into the power system becomes substantial, and may start to influence the dynamic behaviour of the system [28].

The integration of new technologies of EG can significantly affect all types of stability (e.g.: angle, frequency and voltage stability). Ensuring a distribution network keeps stable under normal and emergency conditions is becoming a more complicated problem.

3.2 Power Flow

The classical method for stability analysis involves some simplifying assumptions. Transmission lines and transformers are simplified as a network of shunt and series reactances. Synchronous generators are represented as a voltage source in series with an inductive reactance, which can be assigned one of three values (called synchronous, transient, or subtransient) depending on the expected timescale of the fault event.

The relationship between voltage and current flows in a network can be represented by node equations. The network equations in terms of the node admittance matrix can be written as follows:

$$\begin{bmatrix} \tilde{I}_1 \\ \tilde{I}_2 \\ \cdots \\ \tilde{I}_n \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} & \cdots & Y_{1n} \\ Y_{21} & Y_{22} & \cdots & Y_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ Y_{n1} & Y_{n2} & \cdots & Y_{nn} \end{bmatrix} \begin{bmatrix} \tilde{V}_1 \\ \tilde{V}_2 \\ \cdots \\ \tilde{V}_n \end{bmatrix} \tag{3.1}$$

where

n is the total number of nodes

Y_{ii} is the self admittance of node i

(sum of all admittances terminating at node i)

Y_{ij} is mutual admittance between nodes i and j

(negative of the sum of all admittances between nodes i and j)

$\tilde{V}_i$ is the phasor voltage to ground at node i

$\tilde{I}_i$ is the phasor current flowing into the network at node i

With these equations, a steady-state solution (i.e. Newton-Raphson method) can be performed to find power flows and phase angles in normal operation.

3.3 Frequency Stability

3.3.1 Introduction

System frequency f is a continuously changing variable and is determined by active power balance between demand and generation. If demand is greater than generation, frequency will drop; if generation is greater than demand, frequency will increase. If there is not enough reserve generation, a sudden increase in demand or a failure of generation will be able to cause a large frequency variation, or even frequency collapse. In this case, widespread demand may have to be disconnected until the frequency recovers [29]. To avoid unacceptable drop in frequency, it is necessary to have enough available reserve generation which can be called upon at notice within seconds or minutes [9].

3.3.2 Active Power and Frequency Control

In the UK, the electricity system operates at a nominal frequency of 50Hz. The system frequency should be kept close to this nominal value for satisfactory operation of the power system [8]. The reasons for keeping the constancy of frequency are:

- Constancy of frequency ensures the constancy of induction and synchronous motor speed, which is particularly important for satisfactory performance of generating units. The performance of a generating unit is highly dependent on the performance of its auxiliary drives, which are associated with fuel, feed water and combustion air supply, etc. When the frequency is low, the motor speed will decrease, hence the auxiliary drives will also decrease. This will affect the generation of the power plant and lead to a further lack of real power and consequently lower frequency.

- Electric clocks and frequency applications for other timing purpose require accurate maintenance of synchronous time, which is proportional to the integral of frequency. The decrease of frequency may affect the precisions of electric clocks.

- A large variation in frequency may also affect the proper system functioning and result in protection devices disconnecting generators or loads [11].

The frequency control can be achieved by scheduling generation to match the demand, using speed governors on each generating unit, and allocating production to generators by central control. These control actions can reduce the undesirable variations in system frequency caused by the changes in real power demand of the system.

3.3.3 Embedded Generation Impact on Frequency Stability

A significant part of EGs are renewable energy sources. The output of some renewable generators, such as wind generators, are not entirely controllable due to their fluctuating sources. The most credible control for this is output constraint [29]. Increasing the penetration of uncontrolled generators makes it more difficult to keep the system balanced. In the case of a large wind farm, fast wind changes and very high wind speeds (wind speeds exceeding the cut-out wind speed) may result in the sudden loss of generation [27]. A significant unbalance between generation and demand would lead to a large deviation of system frequency and dynamically unstable situations [11].

Network conditions such as a network failure or a conventional generator breaking down, may lead to a dropout of a large number of embedded wind generators, since the frequency protection relays of the wind farms might be easily activated by frequency oscillations. This could cause a major lack of generation, which leads to a drop in frequency and may eventually collapse the system [27].

3.4 Voltage Stability

3.4.1 Introduction

Voltage instability happens when there is a progressive and uncontrollable decline in voltage caused by disturbance. The main factor causing instability is the inability of a power system to meet the demand for reactive power.

3.4.2 Reactive Power and Voltage Control

Voltage at the terminals of all equipment should be constrained within acceptable values. Equipment is designed to operate at a certain voltage rating. Voltages outside this range may be harmful or even damage the equipment performance [8].

Reactive power flow should be minimised to reduce losses. This ensures the transmission system operates efficiently in transmitting active power. Power Factor (PF) correction devices, such as shunt capacitors, are widely used to correct the power factor and reduce reactive power demand. These devices could be implemented at central substations, distribution system, or built into power consuming equipment. Since reactive power cannot be transmitted over long distances, they should be implemented throughout the system.

3.4.3 Embedded Generation Impact On Reactive Power and Voltage Control

When large numbers of EG units, each of which has its own control system, are connected to a network, their behaviour and their interactions with other machines and other control algorithms are not predictable.

Wind generators are suggested to have less ability to control the grid voltage than conventional generators [11]. The conventional synchronous generator has a capability, which generally provides a constant reactive power output when the real power output is below rated value. Wind generators like DFIG do not exhibit the same characteristics and induction generators need to absorb reactive power. In the event of transient instability, a synchronous generator will pole-slip. An induction generator, however, will draw large reactive currents at over speed. This will decrease the network voltage further and may lead to voltage instability [30].

For wind generators under power factor control, as the wind fluctuates, real power generation will fluctuate and the reactive power generation will also fluctuate to maintain a constant power factor. Fluctuations of power output will lead to changes in voltage [27].

Conventional control systems were designed for conventional centralised networks with no considerations on the power flow changes caused by EG. When they are applied to EG, the system might not operate as expected. Also, the control algorithms were designed for the system operator, receiving data from the whole national grid and controlling all the generators. Serious difficulties might be encountered due to the large number of generators involved [29].

3.5 Transient Stability

3.5.1 Introduction

Transient stability is the ability of the power system to maintain the synchronism of interconnected synchronous generators after being affected by a transient disturbance (e.g.: short-circuit on a transmission line). It depends on the ability to maintain or restore equilibrium between electromagnetic torque and mechanical torque of each synchronous machine in the system [8].

When a disturbance occurs, variables in the system, such as generator rotor angles, power flows and bus voltages, start to deviate from their steady state values. If the resulting oscillations in these variables are able to damp out and the system eventually restores a state of operating equilibrium, it will be regarded as stable. Whereas if the angular swings of some synchronous generators increase and lead to their loss of synchronism with other generators, the power system will become instable.

3.5.2 Embedded Generation Impact on Transient Stability

At the early stage of EG development, the impact of EG on transient stability was too small to be taken into account. Hence it has usually been considered as negative load [10]. With its penetration level increasing, the contribution of EG to power system stability may become significant depending on the capacity and electrical characteristics of embedded generators. The generators in the distribution network may interact with generators in the transmission network and alter the transient characteristics of the power system [5].

Unlike the directly coupled conventional generators, some embedded wind generators are coupled to the grid through inverters. Hence they may interact with the power system in a different way due to their different characteristics, and have different responses to a disturbance [11].

3.6 Protection

3.6.1 Introduction

The goal of protection device is to prevent components in the power system being damaged by fault currents, over-voltages or over-speed etc. Protection devices work when a certain variable exceeds a threshold value, which has been set in advance and stored in the device.

A protection system should be properly configured, or else a fault may trigger a chain reaction and lead to more equipment being tripped out than is necessary to clear the fault [29].

3.6.2 Embedded Generation Impact on Protection

When a fault occurs on a distribution network to depress the network voltage, the EG may overspeed and be tripped out by its internal protection. As EG plant usually has a low inertia and the tripping time of distribution protection is often long, it would be more difficult to ensure the stability for all faults on the distribution network [30].

3.7 Summary

This chapter introduced some common stability problems of power system, such as transient, frequency and voltage stability.

Traditionally, the distribution network is passive and remains stable under most circumstances. With the penetration level of EG increasing, its influence on power system stability cannot be neglected any more. The impacts of EG on power system stability, due to its different characteristics from conventional generation, have been explained in this chapter. Large amount of EG units being implemented in the distribution network can significantly affect all types of stability. It becomes more difficult to ensure distribution network stability under normal and emergency conditions.

The fluctuating nature of renewable sources and their sensitive protection devices may affect the frequency stability of the power system. Also, renewable generators may have less ability to control the grid voltage than conventional generators. Voltage stability problems may also be caused by misuse of the control system for EG. Furthermore, EG units may behave differently from conventional generators and impact on power system transient stability.

Such understanding of EG impact helps in finding proper approaches for deriving dynamic equivalents of a distribution network with EG.

Chapter 4

Dynamic Equivalencing

4.1 Introduction

In order to guarantee operational reliability, the accuracy of power system stability studies must be satisfied. The dynamic behaviour of the power system should be estimated accurately, studied and analysed with accurate power system models.

Factors like their large size, nonlinearity and operational uncertainty ensure the complexity of inter-connected power systems. Over recent years, there has been a trend towards larger and more complex power systems. Due to the large number of transmission lines and system components that a power system can have and the significant neighbourhood network influence on it, it may become more difficult to perform the dynamic analysis and stability problem studies accurately by modelling the power system in a detailed way. Also, the data available on power system behaviour is usually restricted because of the limited cooperation caused by the competition.

To meet the requirement of developing proper and efficient models for a large scale and complex power system, many studies have been carried out on power system dynamic equivalencing, which is used to find a reduced dynamic representation (i.e. dynamic equivalent model) of part of the power system and use it to replace the original part.

Dynamic equivalencing can ideally simplify the original power system model whilst keeping the accuracy in simulating its dynamic behaviours. The resultant dynamic equivalent models can be utilized in dynamic analysis and stability studies for power system reliability analysis, power system operation management, planning to avoid blackout situations and to cope with any new technical circumstances. The application of dynamic equivalents could reduce the model implementation difficulty and computational effort by properly reducing the dimension of the power system model.

4.1.1 Reasons for Dynamic Equivalencing

Studies on dynamic equivalencing have been carried out for many years. The reasons why it has attracted much attention are given below [31]:

- Increasing complexity

  A power system could consist hundreds of machines. The implementation of some new components which have important dynamic characteristics (e.g.: embedded generation and power electronic converters) [32], and the implementation of control equipment (e.g.: power system stabilisers), excitors and governors would increase the system order to a very high value. The increasing complexity may lead to the difficulty in dynamic analysis and stability problem studies with the detailed model of a large power systems.

- Data availability

  The model of an entire electric power system, which involves all the detailed neighbourhood networks, is not always available. In some countries, different utilities hold different parts of the power system.

These utilities have their own control centres, which treat other parts of the power system as external systems. Their competition may limit the cooperation between network operators and restrict the data availability. Also, some local network data may not be possibly obtained in detail (e.g.: information of wind turbines, small generating units and control systems). Some local distribution network operators would want to keep their specific data (e.g.: the capability of power plants and the performance of loads) undisclosed. As mentioned in 2.4.4, the outputs of embedded generation power stations cannot be directly seen by the transmission system operator (National Grid) and are outside its control. What the transmission system operator can see is the national system frequency. The detailed information of EG is voluntarily sourced by the distribution network operators and hence may have not been updated adequately or guaranteed.

- Computational difficulty

  Even if the data of the entire system were available, it would be difficult and expensive to maintain the relevant database. The calculation of a detailed entire large power system would lead to computational difficulty and long computational time. It would also be limited by the size of computer memory [32], especially in real time applications, where sensing and computational resources could be limited.

- Software limitation

  In addition, some new components in the power system might not yet have been included in the standard model library of some commercial software packages. This would cause difficulty in accurately modelling these components. Also, most commercial power system simulation packages for transient stability studies have their maximum system size constraints. This would make it impossible to model an entire power system in detail due to its large size and complexity.

- Necessity

  In some applications, such as designing controllers for the power system, a simplified dynamic model of the power system is usually applicable. Some parts of the power system, which are far away from the disturbances may have relatively small effect on the system dynamics and are not of direct interest. Although their impacts on the rest part of the power system during the disturbance period needs to be taken into consideration, it is not necessary to model them in detail or with great accuracy.

For these reasons, dynamic equivalencing has been developed to simplify the power system model and keep reasonable accuracy at the same time.

4.1.2 Concept of Dynamic Equivalencing

A complex interconnected power system can be divided into two systems:

- The study (internal) system

  The study system is a part of the power system, whose response is of direct interest and where all the disturbances and configuration changes are assumed to happen. Hence it should be retained in detail containing all the internal machines for dynamic analysis and stability studies.

- The external system

  The external system is the rest of the power system, which contains external generators, transformers, transmission lines etc. It can be simplified into a reduced equivalent since we are not interested in the external system itself but its effects on the study system.

Dynamic equivalencing is a process of reducing the order of the external power system model. The obtained dynamic equivalent is used to represent the external system when the study system is affected by disturbances. The key issue of a dynamic equivalent is its ability to indicate the trend of the external system dynamics. The dynamic response of the power system with the external system being replaced by dynamic equivalent should be as close as possible to that with the original external system model. In the process of deriving a dynamic equivalent, it is always important to preserve the original dynamic characteristics.

4.1.3 State-of-the-art for Dynamic Equivalencing

Many studies have been carried out to create dynamic equivalents with various degrees of complexity of the power system which has helped in improve the efficiency of dynamic analysis and stability studies. The initial works in this area are empirical based methods, which use the idea proposed by Ward [33][34] to reduce the nodes of the network. Later dynamic equivalencing is based mainly on modal analysis. Early work in dynamic equivalencing includes the development of model reduction techniques.

The coherency-based approach is based on the idea that coherent generators swing together in transient periods. It was first proposed by Podmore [35] in the late 1970s and has been integrated in the dynamic equivalencing software package, DYNamic REDuction (DYNRED), developed by the Electric Power Research Institute (EPRI) [36]. Evaluation of performance of some coherency-based dynamic equivalencing techniques and discussion of factors that could affect the quality of dynamic equivalents are given in [37][36]. There are also some works in the literature concentrating on combining the theoretical basis of the coherency-based approach and modal analysis. Under some conditions, conventional equivalencing can benefit from working jointly with alternative approaches.

More recently, the system identification approach was proposed. New alternative techniques such as Artificial Neural Network (ANN) based system identification have appeared. System identification methods are useful as they could derive dynamic equivalents directly from measurements.

Hence the existing dynamic equivalencing approaches can be divided into the following categories, Ward approach, modal analysis, coherency-based approach and system identification. A general overview of these approaches is presented in the following sections.

4.2 Ward Equivalencing

4.2.1 Introduction

In the external power system, some buses, which are not strongly affected by the disturbances, can be reduced. The conventional node approach is based on the Ward equivalencing technique to eliminate these selected nodes of the network [38].

4.2.2 Method Description

The Static Ward Equivalent

For the original power system, the bus current $\bar{I}$ is related to the bus voltage $\bar{U}$ and the bus admittance matrices $\bar{Y}$ through [39]:

$$\begin{bmatrix} \bar{I}_g \\ \bar{I}_l \end{bmatrix} = \begin{bmatrix} \bar{Y}_{gg} & \bar{Y}_{gl} \\ \bar{Y}_{lg} & \bar{Y}_{ll} \end{bmatrix} \begin{bmatrix} \bar{E}_g \\ \bar{V}_l \end{bmatrix} \tag{4.1}$$

Where
The subscript l denotes the load buses to be eliminated.
The subscript g denotes the generator buses that are retained.

When the system is reduced to the internal generator nodes, the current-voltage relationships are reduced to:

$$\bar{I}_g^{eq} = \bar{Y}_{eq}\bar{E}_g \tag{4.2}$$

Where

$$\bar{I}_g^{eq} = \bar{I}_g - \bar{Y}_{gl}\bar{Y}_{ll}^{-1}\bar{I}_l \tag{4.3}$$

denotes the equivalent current injection vector and

$$\bar{Y}_{gg}^{eq} = \bar{Y}_{gg} - \bar{Y}_{gl}\bar{Y}_{ll}^{-1}\bar{I}_{lg} \tag{4.4}$$

denotes the equivalent bus admittance matrix [34].

Dynamic Ward Equivalent

However, the static Ward equivalent of a power system with the voltage-dependent loads is only accurate around the operating point at which it was computed. When the operating point shifts, the model may not be able to represent the power system properly since the equivalent currents at the generator buses are not valid any more.

To solve this problem, the dynamic Ward equivalencing approach was developed on the basis of static Ward equivalencing. A proper correction formula, which allows the update of equivalent current injections at the retained nodes, has been used as follows [39]:

$$\bar{I}_g^{eq} = \bar{I}_{g0}^{eq} + \Delta\bar{I}_g^{eq} \tag{4.5}$$

Where

The subscript 0 refers to the quantity associated with the base case operating point.

$\Delta\bar{I}_g^{eq}$ denotes the incremental changes in the equivalent current injections resulting from the shift in generator angles.

By substituting $\bar{I}_g$ from (4.1) into (4.3) we can obtain:

$$\Delta\bar{I}_g^{eq} = \bar{Y}_{gg}\Delta\bar{E}_g + \bar{Y}_{gl}\Delta\bar{V}_l - \bar{Y}_{gl}\bar{Y}_{ll}^{-1}\Delta\bar{I}_l \tag{4.6}$$

Where

$$\Delta E_{gi} = E_{gi} - E_{g0i},\ i = 1, \cdots, n \tag{4.7}$$

$$\Delta V_{lj} = E_{gi} - E_{l0j},\ j = 1, \cdots, m \tag{4.8}$$

$$\Delta I_{lj} = E_{gi} - E_{l0j},\ j = 1, \cdots, m \tag{4.9}$$

Linearisation is carried out to relate the incremental phasor vectors $\Delta\bar{E}_g$, $\Delta\bar{V}_l$ and $\Delta\bar{I}_l$ to the machine angle increments $\Delta\delta_g$ and derives the expression of the equivalent current increments as a function of $\Delta\delta_g$ and a sensitivity matrix $\bar{W}$:

$$\Delta\bar{I}_g^{eq} = \bar{W}\Delta\delta_g \tag{4.10}$$

The swing equation of synchronous machine is given as:

$$M_i\ddot{\delta}_{gi} + D_i\dot{\delta}_{gi} = P_{mi} - P_{ei} \tag{4.11}$$

Where

P_{ei} is the generator electric power obtained from a load flow solution.

P_{mi} is the mechanical power.

When the system is reduced to the internal generator nodes, the equivalent electric powers and their increments are found through:

$$P_{e0i}^{eq} = E_{g0i} \sum_{j=1}^{n} E_{g0j} Y_{ij}^{eq} cos(\delta_{g0i} - \delta_{g0j} - v_{ij}^{eq}) \tag{4.12}$$

$$\Delta P_{ei}^{eq} = E_{g0i} \Delta I_{gj}^{eq} cos(\delta_{g0i} - \Delta \Psi_{gi}^{eq}) \tag{4.13}$$

$$P_{ei}^{eq} = P_{e0i}^{eq} + \Delta P_{ei}^{eq} \tag{4.14}$$

where

$Y_{ij}^{eq} \angle v_{ij}^{eq}$ are the entires of the equivalent bus admittance matrice Y_{gg}^{eq} as expressed in Equation (4.4).

$\Delta I_{gi}^{eq} \angle \Delta \Psi_{gi}^{eq}$ are the equivalent current increments updated by the sensitivity matrix as expressed in Equation (4.10).

4.2.3 Method Evaluation

For static Ward equivalencing, if all the generator nodes in the power system are retained, the obtained equivalent is called Ward-PV equivalent. Since this equivalent contains all the generator nodes, it may also be applied to dynamic analysis [38].

The Ward-PV equivalent may retain a large number of generator nodes. Machowski [38] has developed a reduced Ward-PV equivalencing approach. By grouping and aggregation of generator nodes, each selected group of generators is represented by a single equivalent generator node.

In [39], Baldwin proposed a sensitivity-analysis-based fast dynamic Ward equivalent. In this method, the equivalent current injections are expressed based on the retained bus angles and a sensitivity matrix E. This technique uses a correction formula derived from sensitivity analysis, which updates the equivalent current injections at the retained nodes without running any load flow calculations. The sensitivity matrix is updated when the machine angles shift beyond the range of validity of the linearisation.

In [40], the Ward-PV approach was used to work with other dynamic equivalencing approaches. Once equivalent generators are derived by other dynamic equivalencing methods and the reduced external system contains only the equivalent generators and the original loads, the Ward-PV approach has been applied to eliminate the selected load nodes of the external system.

Advantages

The Ward equivalencing approach has the following advantages:

- It can provide accurate results when it is applied to a linear passive network [38][40].
- The dynamic Ward equivalencing approach can be used for transient stability analysis.

Disadvantages

The downsides of the Ward equivalencing approach are listed below:

- The Ward approach can be used for eliminating load nodes (PQ buses) but is not suitable for eliminating generator nodes (PV buses) since it cannot represent the reactive capabilities of PV buses under disturbance conditions [34]. Also, for transient stability analysis, PV buses would require the original generator data to be decomposed to create the dynamic data, which would increase the order of the model [41].
- It is difficult to find the undisturbed buses in the power system, which are selected to be eliminated [42].
- Another major drawback of Ward type equivalent is its unreliability caused by over-simplification of the load models [39].

- The Ward equivalencing approach is based on the linearised differential equations of the generator rotor movement [43].
- Dynamic Ward equivalencing could be time consuming, which precludes its use in real time environment.

4.3 Modal Analysis

4.3.1 Introduction

The modal analysis approach is based on the concept that the modes not easily affected by disturbances in the power system can be eliminated. This approach uses the linearised model of the external system and its eigenvalues to reduce the system order. It is usually used in small signal stability studies but could also be used for large disturbance analysis [44].

The assumptions of this approach are as follows:

- The external system is far away from the disturbance points in the study system and the state changes caused by the disturbance in the external system are small, which makes it reasonable to represent the external system by a linearised model.
- The dynamic characteristics of the external system are described by using voltages and injection currents at the connecting nodes as inputs and outputs respectively, which are linearised at the operating point.

4.3.2 Method Description

The full order model of the external system, linearised at the operating point can be described as a state space model [45]:

$$\Delta \dot{X} = A \Delta X + B \Delta U_T \tag{4.15}$$

$$\Delta I_T = C \Delta X + D \Delta U_T \tag{4.16}$$

where

X are the original state variables.

U_T is the node terminal bus voltage at the connecting bus.

I_T is the node injection current at the connecting bus.

A, B, C, D are the coefficient matrices composed of generator equations, coordinated transformation equations and algebraic equations of the transmission network.

Δ is a small deviation from initial value.

This model can be transformed by diagonalising the system using eigenvalues and eigenvectors, which can be calculated through:

$$T^{-1} A T = \Lambda \tag{4.17}$$

$$X = TY \tag{4.18}$$

$$\begin{aligned} \Lambda &= diag(\lambda_1, \lambda_2, \cdots, \lambda_n) \\ T &= [v_1, v_2, \cdots, v_n] \end{aligned} \tag{4.19}$$

Where

Λ and T are the diagonal matrix of eigenvalues and the matrix containing eigenvectors respectively.

By this transformation, the diagonal canonical form of state equations can be obtained as:

$$\Delta\dot{Y} = \Lambda\Delta Y + B_s\Delta U \quad (4.20)$$

$$\Delta I = C_s\Delta X + D\Delta U \quad (4.21)$$

Where

Y is the transformed state vector on eigenvector basis.

B_s and C_s are the revised matrices of B and C due to diagonalisation of matrix A.

The basic concepts of modal analysis approach are as follows [45][46][44]:

- Aggregation of similar modes
- Elimination of insignificant modes (e.g.: the modes damped fast or at high frequencies)

This reduction procedure will be executed until the mode number is reduced to a specified number [45][43].

Other than the modal analysis method mentioned above, there are also some model order reduction techniques, which are based on directly identifying and preserving certain modes of interest, that can be applied to linear systems. These techniques are listed as follows:

- Singular Value Decomposition (SVD) method
  - Balance truncation [47]

    This method focuses on the observability and controllability of the system. The original state-space model is transfered to a new one, which has each state variable controllable and observable. Weakly controllable and observable states can be truncated for model reduction.

- Hankel norm approximation [48]

  This method tries to achieve a compromise between a small worst case error and a small energy error.

- Singular perturbations [49]

  This method decomposes the system based on the fast and slow dynamics of the system. Fast dynamics can be neglected and its effect will be reintroduced as boundary layer corrections to obtain correct static gains.

- Moment matching (Krylov) method [50]

  This method is an iterative method, which focuses on the leading coefficients of a power series expansion of transfer function. The coefficients of equivalent should match those of the original system around a user-defined point.

4.3.3 Method Evaluation

Modal analysis approach has been commonly used in the early work of power system dynamic equivalencing.

Considering the fact that old model reduction techniques would lead to a set of reduced-order equations, which can not be interpreted as a representation of the same physical system, Tsai [51] represented a Structure Preserving (SP) model reduction technique, which preserves the structure of the physical system in the reduced model. Unlike other modal analysis methods, this method keeps the order of the vector differential equations and reduces the dimension of the vector. Thus the reduced system may have the same mathematical structure and hence the same physical type as the original system representation.

In [52] it is suggested that the dynamics of the power system is dominated by the swing modes, which are poorly damped and related to machine inertias. The excitation and governor system only modify the damping of swing modes. Hence a modal analysis method was proposed to separate the machines associated with swing modes, from the rest. The machines associated with swing modes can form an equivalent of the original power system.

Oliveira [46][53] derived a second-order nonlinear dynamic equivalent with assistance of modal analysis. This method consists of defining the parameters of a modal generator (nonlinear electro-mechanical structure) for each retained mode of oscillation. These modal generators can simulate the dynamic response under small disturbances and also the nonlinear characteristics under large disturbances of the original power system.

In [49] Castro compared several model reduction techniques and suggested that the singular perturbations technique seems to be the most promising one among them. The singular perturbations technique has been applied to wind park dynamic equivalencing. The obtained reduced model is able to keep the associated dynamics of wind park under small perturbations around the steady operating point, which are caused by wind speed fluctuations.

The modal analysis approach normally uses state-space models to describe the power system. Veliz [54] introduced a method that uses a matrix in the s-domain to model the power system for modal analysis. This method is based on computing dominant poles (which have the associated residues with high moduli) and the associated residues for s-domain models of power system. Compared to the state-space model method, the s-domain method has the advantage of modelling frequency-dependent characteristics of the transmission lines and building the nodal admittance matrix (a particular matrix of the s-domain matrix) easily.

Ishchenko has applied some model reduction techniques, such as SVD-based techniques (balanced truncation [47] and Hankel norm approximation

[48]) and Krylov-based technique [50], on a small distribution network with EG for dynamic equivalencing. The dynamic equivalents derived by these model reduction techniques can retain the dynamic behaviour of EG and its impacts on the transmission network. Simulation results suggested that a linear model can provide enough accuracy to represent the distribution network with EG.

Advantages

Modal analysis has the following advantages for dynamic equivalencing:

- It has a strict mathematical basis and can provide a good insight into various modes of oscillations in the system [55][44].
- It has the ability to control the size of the dynamic equivalent in a systematic manner [55]. The external system can be highly simplified while the dominant eigenvalues are kept [44].
- The quality of the dynamic equivalent derived by modal analysis does not depend on the disturbances. Fault tests are not required for the original power system to construct dynamic equivalents [55].

Disadvantages

Modal analysis has the following disadvantages for dynamic equivalencing:

- The major downside of the approach is that its computational task may cause difficulty in practical applications. With the size and complexity of the power system increasing, eigenvalue techniques may not be applicable any more for stability studies since it would become difficult to evaluate eigenvectors.
- The dynamic equivalent derived by the modal analysis approach may have an abstract nature (i.e. described in the form of the

linear state function) rather than a composition of actual physical components [55][43]. Data conversions will be required when the dynamic equivalent needs to be implemented into the simulation program for transient stability analysis, or modifications have to be made in the original dynamic simulation programs to make use of the state matrix form of the dynamic equivalent model [42].

- The modal analysis approach can only be applied to external systems, which are linear or can be linearised around an operating point. Therefore, the approach has an inherent limitation since linearisation of the model is required [55]. When the linear methods cannot properly capture the complex dynamics of the power system, especially during major disturbances, it could present difficulties in reducing the orders of the non-linear models. Also, the derived dynamic equivalent may not be able to retain the non-linear characteristics of the system [43].

- Elimination of modes is based on empirical methods. An improper elimination may lead to instability of the power system model during stability studies. Difficulties will arise if the engineer has not got the experience to make decisions in the elimination procedure.

- The approach requires the complete information of the original power system to be known in advance, including the parameters of all the components in it. Such availability of data may not be achievable for a large inter-connected complex power system.

4.4 Coherency-Based Equivalencing

4.4.1 Introduction

Coherency is an observed phenomenon in an interconnected power system, where a certain group of generators tend to swing together after a disturbance. These generators can be aggregated into a single equivalent generator since they have coherent transient responses to a particular disturbance. Coherency properties between generators are associated with the type and the location of the disturbance.

The coherency-based dynamic equivalencing includes three steps:

- Identification of coherent generators
- Aggregation of nodes
- Aggregation of generators and control devices

4.4.2 Identification of Coherent Generators

Coherency identification of generators is the key step of coherency-based dynamic equivalencing, which affects the quality of dynamic equivalent most.

The early attempts to identify the coherency of generators are heuristically-based methods. In [56] Wu presented a physical interpretation of the algebraic characterisation of coherency. The condition of coherency is described in terms of inertia constants and the equivalent admittances to where the disturbance happened (electrical distances). Such heuristically-based methods lack accuracy and consistency.

In [57] Gallai suggested that within a certain range, the coherency properties of a non-linear power system are in accordance with its corresponding linearised model. Hence in the following studies, most coherency identification methods are working on the linearised power system model. Coherency of generators depends on the structure of the power system, generator inertia constants, generator damping constants and the location of the disturbance.

Later, a method initially proposed by Podmore [35] became a common approach to coherency-based dynamic equivalencing and has been successfully used in reducing the size of power system model significantly (e.g.: use in the DYNRED [36] program). This coherency-based approach works on a simplified and linearised model of power system and uses a clustering algorithm to process the swing curves to determine the coherent generator groups.

Classical Coherency Identification

The classical way to identify coherency is by processing the obtained swing curves after a particular disturbance [35]. If the phase angular difference between generator i and generator j has a constant value during the simulation, it will suggest that the two generators are electrically coherent.

The simplified coherency condition can be described as [31]:

$$\delta_i(t) - \delta_j(t) = \delta_{ij}(0) = constant \tag{4.22}$$

Where

δ_i is the rotor angle of generator i

δ_j is the rotor angle of generator j

δ_{ij} is the phase angular difference between the two generators.

Many studies have been done using different coherency identification techniques, such as frequency response [58], relation factor [59], Taylor series expansion [60] and fuzzy clustering algorithms [61]. Most of them work on the linearised power system model and use various coherency identification criteria to determine the coherent generator groups without solving swing equations. The procedure includes defining a coherency measure and applying the suitable clustering algorithms [62]. Various variables of the generators can be used as measures to determine the coherency, such as rotor angle [42][58], rotor angular speed [62], rotor angular speed deviation [63], rotor angular speed and acceleration [64][65] and mode shape [66].

Other than the classical coherency identification approach, which studies the swing curves of generators after a disturbance, there are also some analytical approaches, which evaluate the generator coherency independently of disturbances, such as weak link coupling and slow coherency approaches [67][68].

Weak Links Coherency Identification

In [69], Nath proposed a weak links method for large power systems, which determines the coherency by analysing the coupling of generators directly from the state matrix of the power system [59]. This method identifies the weakly coupled areas to divide the power system into study and external systems. The coherent group can be identified by finding the tight tolerance band group where the coupling coefficients among the generators are high.

Slow Coherency Identification

Time domain simulations may require considerable computational efforts and the identified coherent groups are dependent on the disturbance. To solve this problem, a two-time scale method, or slow coherency method, was proposed and became a commonly used method for coherency identification [70]. Slow coherency method is based on modal analysis (i.e. singular perturbations theory) to identify the coherent machines [38][58]. It has been included in the DYNRED program [36] developed by EPRI [71].

In [67], Yusof suggested that the dynamics of the power system can be grouped into slow and fast modes. The slow coherency approach is based on the concept that the slow modes (at lower frequency) of oscillations are caused by two strongly coherent generator groups inter-connected through weak ties (interarea modes). The coherency of generators can be identified by means of the eigenvector associated with the mode of oscillation. The power system can be partitioned into coherent groups by selecting the slowest modes, the number of which determine the number of the coherent groups [72]. Each group is represented by a reference generator, which is corresponding to the most linearly independent rows of the slow eigenbasis matrix. Each non-reference generator is grouped to its reference generator according to the highest coherency factor.

Ramaswamy [73][74][75][76] proposed a Synchronic Modal Equivalencing (SME) method, which is based on selective modal analysis and slow coherency. This method divides the power system into internal and external system on the basis of slow coherency and modally equivalences the generators in the external system using selective modal analysis. The derived dynamic equivalent is in the form of linear muti-port admittance, which can be easily represented by controlled current injectors at the connecting bus.

In [37] Chow presented a tolerance-based slow coherency method, which includes additional constraints to ensure that the widely separated generators are not aggregated. Joo [77] reported a tolerance-based slow coherency technique using a K-means algorithm to identify coherent machines. Unlike the previous tolerance-based slow coherency method, this method involves the initial system operating conditions for coherency measure. The overall performance index is proposed in the paper to avoid the difficulty in selecting proper tolerance values for tolerant-based coherency method.

Most dynamic equivalencing methods require the complete information of the power system to reduce part of it. However, a power system can be interconnected with another power system, whose information is not always available. In a recent paper [78], Marinescu introduced a border synchrony method, which is based on SME and balanced realisation techniques. SME is used to derive a structure-preserving equivalent, which retains the interarea modes between two interconnected power systems. These modes are identified from a balanced realisation of the power system, which needs to be reduced, using no data from the power system it is interconnected with.

4.4.3 Aggregation of Nodes

The next step is to link the terminal buses of coherent generators to a single node. The coherent generator group will be replaced by a single equivalent generator connected to this node. This equivalent generator must have similar behaviour to the original group of generators.

Aggregation of Generating Nodes

Zhukov's method has been used to aggregate the generating nodes in coherent generator groups [31]. In nodal aggregation, tie lines between the external system and study system remain unchanged. An equivalent generating node is created to replace nodes of the coherent generators. It should satisfy the following two conditions [79]:

1. The currents and the voltages at the tie buses do not change.
2. The total power injected to the equivalent node equals to the sum of

the power injections at the terminal buses of coherent generators.

$$P_{eq} + jQ_{eq} = \sum_{i=1}^{n} (P_i + jQ_i)$$

Chow [71] suggested that since machine coherency is identified by machine rotor angular swings, it is more appropriate to do the aggregation at the machine internal node rather than its terminal bus. In this paper, coherent generators are connected at their internal nodes, which are linked to infinite admittance. This internal node aggregation method notes the fact that machine internal nodes are not connected with infinite admittance into consideration and hence introduced an impedance correction.

Aggregation of Load Nodes

The derived equivalent should satisfy the following conditions [80]:

1. The total load injected into the equivalent node should be equal to that injected into the original system.
2. The voltage of the equivalent node should be equal to the weighted mean of the aggregated nodes.
3. The voltage of a retained node should be the same as that before reduction.

The load nodes can be aggregated into a few equivalent nodes using Dimo's method [31]. Firstly some fictitious branches are added to connect the nodes to be aggregated with a fictitious node, which has a voltage equal to zero. Each branch i has admittance Y_i corresponding to the node injections at a given voltage in the aggregated nodes:

$$Y_i = \frac{S_i}{V_i^2}$$

An extra fictitious branch with negative admittance Y_a is added to connect this fictitious node with the equivalent node, which makes the voltage at the equivalent node V_a equal to the weighted average of the voltages at the aggregated nodes.

$$Y_a = -\frac{\sum S_i}{V_a^2}$$

Then the fictitious node and the nodes to be aggregated are eliminated. An equivalent network connecting the equivalent node with the retained nodes remains.

There are also some load node aggregation methods [81][82][80], which represent the load by induction motor models rather than constant admittances. In [80], an aggregation method for voltage stability studies is introduced. For voltage stability studies, more attention is paid to load than to generators and more to voltage magnitude than to the angle. If the voltage magnitude difference of two load nodes keeps constant within a certain tolerance, they would be defined as voltage coherent and hence reduced into an equivalent node.

Other than using Dimo's method to aggregate the load nodes, node reduction can also rely on Ward equivalencing technique to eliminate the load nodes (see Section 4.2).

4.4.4 Aggregation of Generators and Control Devices

The group of identified coherent generators are then aggregated and replaced by a single equivalent generator connected in parallel to the connecting bus. Generator aggregation refers to the method that modifies the network equations to replace the coherent generator group. Once the system coherent groups have been identified, generators in the same coherent group can be aggregated to one or a few equivalent generators. The generator aggregation can be classified into two forms: classical aggregation and detailed aggregation [36].

In classical aggregation the group of coherent generators is replaced by an equivalent classical model of synchronous generators. The equivalent generator's inertia H_{eq} is calculated with the sum of inertia with respect to the equivalent sum of their apparent power S [79].

$$H_{eq} = \frac{H_1S_1 + H_2S_2 + ... + H_nS_n}{S_{eq}} \tag{4.23}$$

The transient reactance of the equivalent generator can be obtained by paralleling transient reactances of all the coherent generators [36][42]. In [79], the equivalent admittances are suggested to be calculated by paralleling the coherent generators as follows:

$$Y''_{eq} = Y''_1 + Y''_2 + ... + Y''_n \tag{4.24}$$

$$Y'_{eq} = Y'_1 + Y'_2 + ... + Y'_n \tag{4.25}$$

$$Y_{eq} = Y_1 + Y_2 + ... + Y_n \tag{4.26}$$

where Y''_{eq}, Y'_{eq} and Y_{eq} correspond to sub-transient, transient and steady state behaviours.

In detailed aggregation, if generators in a coherent group have similar control systems, they can be aggregated to a detailed generator model with an equivalent exciter, stabiliser and governor [36]. The parameters of this equivalent machine can be obtained by matching its frequency or time domain response to the original ones. The principle of linear aggregation can be used in the aggregation of exciter [72], stabiliser and governor [83].

Galarza [84] proposed an exciter aggregation method, which is used to tune the aggregated exciter parameters of the equivalent model. This sensitivity method determines the optimal parameters for the aggregated exciter model. Some simple rules for computing aggregated exciter parameters, which closely approximate the optimal parameters, are developed. Kim

[40] suggested that if the control units in the coherent group have similar characteristics, the control unit of the reference generator could be used as the equivalent control unit.

Most aggregation methods use iterative procedures to calculate the parameters of the equivalent generator and their control systems in the frequency domain. In [72], a detailed aggregation was developed based on structure preservation of the coefficient matrices in the time domain representation of generators. This method involves less computations than the iterative methods, which use mainly iterative procedures in the frequency domain.

4.4.5 Advantages

The coherency-based dynamic equivalencing approach has the following advantages:

- The obtained dynamic equivalent is based on a nonlinear generator model. Hence it can be compatible with other components in the system and easily implemented in conventional simulation programs for validation purposes [55]. It is obtained from aggregation and is described by physical components similar to the replaced generators but with new parameters.
- It is a nonlinear approach which can capture the complex dynamics of the power system and provide desirable performance in the event of major disturbance rather than under small signal conditions only. The retained nonlinear characteristics can extend the validity of the coherency equivalent.
- The approach is simple but has been very successful in significantly reducing the size of interconnected power system dynamic models in transient stability studies.

4.4.6 Disadvantages

The coherency-based dynamic equivalencing approach has the following disadvantages:

- The coherency between generators is dependent on the selection of disturbance injected into the power system. The formation of coherent groups depends on the nature and location of the disturbance. As a result, the quality of the equivalent is dependent on the disturbances chosen to determine the coherency [55]. Since the technique is very empirical, it is difficult for people who have limited experience to choose an appropriate disturbance. And the confidence in the dynamic equivalent for other disturbances may be very limited [37].
- Complete knowledge of the dynamic behaviour of the original power system is required beforehand to define the coherent groups and perform the aggregation process. Complete parameters of the generators in the external system, which is not always available in reality, are required.
- With the spread of EG units, the coherency-based approach, which depends on analysing the electro-mechanical behaviour of generators through angular speeds or rotor angles, will not be suitable for the following reasons:
  - Some EG units are based on induction generators, which do not have synchronising torques.
  - Some EG units are linked to the network through inverter interfaces.
  - Some EG units, such as fuel cells and photovoltaics, are not characterised by angles or speeds
- Computational effort is required to select eigenvalues and eigenvectors of the full system if slow coherency method is used [37]. The slow

coherency method has difficulty in application to large scale and nonlinear time domain simulations [42].

- The aggregation of nodes in the coherency-based approach may introduce a branch with negative admittance to the power system model. Also, large nodal injections in the aggregated nodes may lead to large resistance values in the equivalent branch. The combination of negative admittance and large resistances may cause problems for load flow programs [68].

4.5 System Identification Equivalencing

4.5.1 Introduction

Conventional dynamic equivalencing techniques, such as modal analysis and coherency-based approaches, require a considerable amount of knowledge about the external system. However, in some cases, the available information may not be enough to develop dynamic equivalents using conventional approaches. To solve this problem, system identification can be used for dynamic equivalencing.

System identification can be used to estimate the dynamic properties of the power system. Unlike the conventional dynamic equivalencing approaches, system identification can derive the dynamic equivalent directly from time domain data, either obtained from measurements or simulation programs [85]. The system identification procedure aims to estimate the parameters for the dynamic equivalent based on measurements of important signals. Hence the parameters and topology of the external power system are not required to be known beforehand. Similar procedures have also been adopted for modelling dynamic loads [86].

4.5.2 Classical System Identification

Classical system identification procedure includes the following steps:

- Firstly, the model structure of dynamic equivalent is selected, such as:
  - Auto-Regressive Moving Average (ARMA) model [87]
  - Auto-Regressive Moving Average with Integrator in the noise model and eXogenous (ARIMAX) model [88][89]
  - State-space model [90][91]
  - Physics-based model (i.e. generator model)[92][93]
- Then the parameters of equivalent model are estimated based on optimisation algorithms, such as:
  - Genetic Algorithms (GA) [94][92][95][96]
  - Quasi-Newton algorithms [88]
  - Levenberge-Marquaridt algorithms [95][88][97]
  - Evolutionary Strategy Algorithms (ESA) [92]
  - Evolutionary particle swarm optimisation algorithms [98]
  - Eigensystem Realization Algorithm (ERA) [90]

From this procedure, a dynamic equivalent can be obtained to replace the original external power system for dynamic analysis and stability studies.

Modal analysis by means of time domain experiments, or Prony analysis, was initially applied to power system by Hauer [99]. Prony analysis is based on the Prony algorithm and can directly estimate the frequency, damping, strength and relative phase of modal components by fitting a weighted sum of exponential terms to a given signal. This feature allows us to obtain the modes of interest directly.

Kamwa [90] proposed a Multi-Input-Multi-Output (MIMO) state space system identification method. This method is based on the system realisation algorithm, which fits a low order state-space model to thw system impulse response. The system realisation algorithm, known also as ERA, is based on SVD of the Hankel matrix associated with system impulse response.

Sanchez-Gasca [85] described a model of the power system based on the product of Hankel matrix and its transpose. The number of the modes to be retained is initially estimated by inspection of the system response and further reduced based on the residual magnitude of the poles.

Miah [100] proposed a method which uses measurement data taken at the connecting bus between the study system and the external system to derive the parameters of the equivalent generator model. The parameters of the original generators in the external system are unknown and the available information is its passive network model and the total inertia constant of the generators in it.

In [91], system identification has been used to model wind turbines. A state space model, rather than a generator model is used as the model structure of a wind turbine. The equivalent of a wind turbine is derived with the measurements of dq-axis voltages and the wind speed as input and the measurements of dq-axis currents as output. As the equivalent of wind turbine is in state-space form, they can form the state-space equivalent of wind parks and are easily integrated into the network nodal equations for Ward equivalencing (see Section 4.2).

In [92][93], Ju presented an online identification method using a linearised third-order generator model. Post-steady response was suggested to be helpful to identify more accurate parameters of the model and the identifiability problem was discussed in depth. This method has been applied to a Chinese system with installed online equipment, using voltage and current as inputs to derive the dynamic equivalent using the data of large disturbances occurring in two years operation. Evolutionary strategies and genetic algorithms are used for the identification procedure.

As mentioned in Section 4.4.2, large-scale interconnected power systems have two time-scale behaviour due to differences in strength of interconnections. The strongly connected generators (coherent generators) can be aggregated to form an equivalent generator. The non-interarea oscillations, or the fast oscillations can be eliminated to reduce the system [101]. In [102][103], Chakrabortty developed an Interarea Model Estimation (IME) algorithm to identify parameters (i.e. reactance and machine inertia) for a two-generator-equivalent of two-area power system (the two areas are weakly connected) from interarea oscillations. These interarea oscillations can be obtained by system identification (e.g.: ERA or Prony) using data from Phasor Measurement Units (PMUs) installed at specific points on the transmission line.

4.5.3 ANN Based System Identification

System identification may not always be able to be expressed in an algorithm or mathematical form [104]. To solve this problem, system identification based on Artificial Neural Network (ANN) has been used.

ANN, which was inspired by how the human brain works, has the ability to learn from the environment and improve its performance by learning [104]. For ANN based dynamic equivalencing, the target dynamic equivalent model does not need to be specified in advance but will be defined through both the structure and the parameter description of ANN (activation functions, biases and weights) [7]. Computer-based equipment is required to be developed and installed in the network for online measurement and identification.

In [32], the procedure of ANN dynamic equivalencing consists of two steps, bottleneck ANN, which is used to extract states of the reduced order equivalent, and recurrent ANN, which is used to predict the new states values of the external system. ANN-based dynamic equivalencing can work with the classic equivalent to reduce the effects of uncertainties and improve the accuracy [105]. The derived dynamic equivalent was implemented into standard software package by being described as a load disturbance waveform.

In [43], the external system is represented by an input-output formulation and only one ANN is used to predict its dynamic behaviour. The nonlinear dynamic external system including all the components such as controllers, is replaced by an ANN interfacing to the study system.

In [97], a recurrent ANN method is introduced and a Series Parallel Model (SPM) structure is chosen to identify the equivalent model. The use of SPM ensures the stability and convergence of the identified equivalent and can provide good performance in nonlinear system identification.

The ANN based method proposed in [7][106] considers the distribution network with a large number of EG units as the external network. The recurrent structure of ANN is used to capture the dynamic behaviour of the external network and interact with the internal network. In this method, current has been used as output instead of power due to its better convergence. Active source and passive loads are modelled separately, which covers different generating and loading conditions in the external system. Hence when the capacity of EG changes, the model does not require re-development.

4.5.4 Advantages

The system identification approach has the following advantages:

- System identification dynamic equivalencing handles the lack of detailed information of the external system and the difficulty caused by modelling a complete large power system. It requires only measurements or simulation from the power system to derive the dynamic equivalent and hence is independent of the network size and complexity.

- The structure of the dynamic equivalent model can be defined by the user. This feature allows the compatibility of the dynamic equivalent with standard models of power system components.

- The ANN-based dynamic equivalent is a nonparametric model, which can keep the nonlinearity characteristics of power system.

- The accuracy of the ANN-based equivalent is not significantly affected by operating point shift and hence is not restricted to certain initial power flow conditions. Once well trained and verified, an ANN based dynamic equivalent can be used in dynamic simulation and control design procedures [7][106].

4.5.5 Disadvantages

The system identification approach has the following downsides:

- Some computer-based equipment is required to be installed in the real power system for the measurements which is then used for system identification.

- Parameters obtained from the measurements may be not unique. Problems such as how to estimate the parameters need to be further studied.

- ANN lacks explanatory capability. It operates as a black box and all dependencies between parameters and responses are hidden.

- It is not possible to convert the ANN structure into known model structures (e.g.: ARMA, state-space), and the interpretation of calculated results is difficult.

- The ANN requires essential time to learn and a large amount of training data to achieve good accuracy. Hence few practical measurements and applications have been reported so far. In the case of power system reconfiguration, an ANN-based dynamic equivalent would need to be retrained.

- ANN-based models may have the risk of over-training, a situation in which the ANN starts to reproduce the noise specific to a particular sample in the training data.

- There is no formal procedure to select the network topology for a ANN-based model. The design of ANN is done experimentally through trial and error, to optimise the number of hidden layers and the number of neurons of these layers according to training performance and prediction accuracy.

4.6 Summary

This chapter overviews the work related to existing dynamic equivalencing approaches. The two main categories of dynamic equivalencing approaches, modal analysis and coherency-based approach, have individual advantages and drawbacks. Several works have aimed to find a proper dynamic equivalent for a distribution network with EG, such as modal analysis methods [47][48][50] or ANN-based system identification [7].

With the increasing complexity of the power system and also new components implemented in the system, the application of modal analysis or coherency-based approach may become limited. Competitive circumstances, a trend towards distributed units and customised control systems make it impossible to obtain the detailed network structure and component parameters beforehand. ANN-based system identification approach also has it own limits such as training time and large data requirements.

Chapter 5

Methodology Description

5.1 Introduction

Figure 5.1 shows the workflow in this project. Figure 5.1 (a) is the real power system where ideally measurements for dynamic equivalencing methodology would be obtained but here simulations are used as a proxy data source; (b) is the original model of the power system, in which the transmission network model is connected to a detailed distribution network model with generators in the system represented by the sixth or fifth order sub-transient models; (c) is the transmission network model connected to the state-space form dynamic equivalent model of the distribution network with the distribution network being replaced; (d) is the transmission network connected with a constant power model.

The dynamic equivalencing methodology illustrated here is based on system identification, which identifies the parameters of a selected model structure based on simulation of the power system. It contains three steps, which are data collection, system identification and model verification.

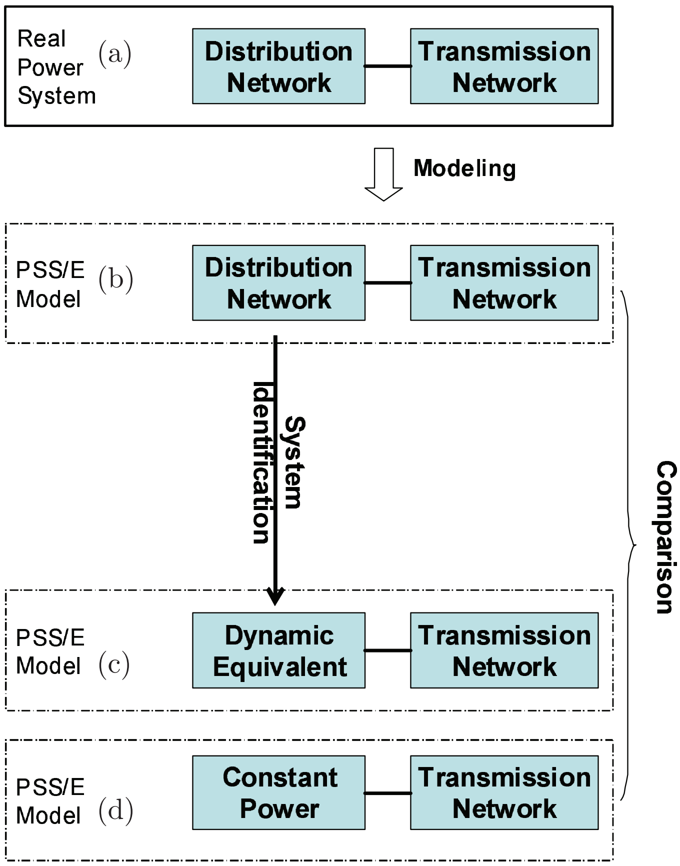


Figure 5.1: Workflow chart: (a) Real transmission network with distribution network ; (b) Transmission network model with detailed distribution network model; (c) Transmission network model with dynamic equivalent model of distribution network; (d) Transmission network with a constant power model

5.2 Data Collection

The commercial package PSS/E program was initially used to simulate the dynamic response of the power system after various disturbances. A dynamic model of the Power system has been implemented which consists of the detailed model of the transmission network and the detailed model of a distribution network (see Figure 5.1(b)).

The response time series is then taken at the connecting bus between the transmission network model and the distribution network model. The time series of real power P, reactive power Q, system frequency f and voltage V have been collected for the purpose of deriving the dynamic equivalent. f and V are used for the inputs and P and Q for the outputs. P and Q will change with V and f based on the voltage and frequency characteristics of the load.

A polynomial model to represent the voltage V and the frequency f dependency of load characteristics is usually represented as:

$$\begin{aligned} P &= P_0 \left[p_1(\tfrac{V}{V_0})^2 + p_2(\tfrac{V}{V_0}) + p_3\right][1 + K_pf(f - f_0)] \\ Q &= Q_0 \left[q_1(\tfrac{V}{V_0})^2 + q_2(\tfrac{V}{V_0}) + q_3\right][1 + K_qf(f - f_0)] \end{aligned} \tag{5.1}$$

where P and Q are the active and reactive components of the load when the bus voltage magnitude is V. V_0 is the the voltage magnitude at the initial operating condition. The parameters of the model are the coefficients p_1 to p_3 and q_1 to q_3, which define the proportion of each component. f is the actual frequency, f_0 is the rated frequency. K_pf, K_qf are the frequency sensitivity parameters.

This model could also be regarded as a voltage dependency model multiplying a factor $[1 + a_f(f - f_0)]$. a_f is the model frequency sensitivity parameter. Since the change in the frequency is insignificant, a polynomial model to represent the voltage dependency of loads is widely used:

$$\begin{aligned} P &= P_0\left[p_1(\frac{V}{V_0})^2 + p_2(\frac{V}{V_0}) + p_3\right] \\ Q &= Q_0\left[q_1(\frac{V}{V_0})^2 + q_2(\frac{V}{V_0}) + q_3\right] \end{aligned} \tag{5.2}$$

This model is commonly referred to as the ZIP model consisting of the sum of the constant impedance (Z), constant current (I) and constant power (P) components [8].

In reality, there are problems in measuring frequency due to the noise. Especially when the disturbance applied to the system is relatively small, the measured signal variation in magnitude could be close to the noise.

The frequency in the power system changes relatively slowly. Therefore in short-term dynamic analysis, the frequency characteristic can be neglected. However in long-term dynamic analysis it should be taken into consideration.

In this project, a method using a single input (i.e. voltage) to derive equivalents has also been tested.

5.3 System Identification

5.3.1 Introduction

System identification is the process of building mathematical models of a complex system based on observed data [107]. Certain assumptions are made for the identification procedure on a given system. Firstly, the system should be observable, which means the time evolution of all the modes in the system of our interest can be reflected at the measured outputs.

Secondly, the system should also be controllable, which means the inputs could excite all the modes of interest [108]. These mathematical models can be described as a box containing mathematical laws, which link the inputs with the outputs of the system.

The solution to the identification problem is not unique. An infinite number of state representations involving different model structures may result in the same observed input-output relationship. They are all equivalents of the system and can be related by similarity transformations [108]. Sometimes difficulty may be encountered in selecting the best equivalent.

In this project, the objective system, which is required to be modeled, is the distribution network with EG. In the system identification step, the collected data for inputs and outputs are imported to Matlab for parameter identification. The system identification process is handled by the Matlab System Identification Toolbox which is developed by Ljung [107] in 1995.

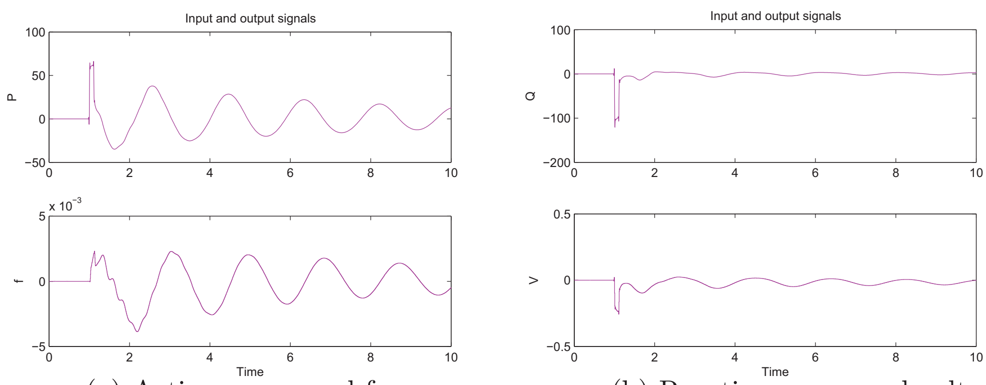
5.3.2 Data Processing

Examine the data

The first step for system identification is to import the input and output data. Figure 5.2 shows an example of imported data in the System Identification Toolbox. Time series of the bus voltage V and the frequency f are the input together with the active power P and reactive power Q injected to the bus for the outputs. They are measured under a simulated three-phase line fault, which is cleared in $0.1s$.

De-trend the Data

The input-output relation could be seen from Figure 5.2. It should be noted that the physical equilibrium offsets (i.e. the steady state values) of these



(a) Active power and frequency (b) Reactive power and voltage

Figure 5.2: Data used for system identification

data have been removed, which is referred to as de-trending. That means the models are describing how the changes of the inputs are affecting the changes of the outputs rather than explaining the actual levels of signals. This data processing operation helps to estimate more accurate linear models because linear models cannot capture arbitrary differences between the input and output signal levels [107]. This is a normal situation in system identification.

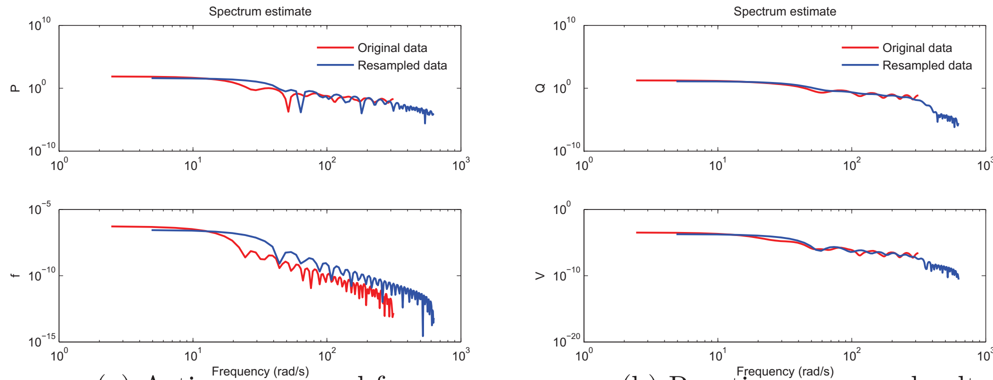
Re-sample the data

As will be explained later in Section 6.3.4, the user defined model in the PSS/E software (which we will use to implement the dynamic equivalent model) is called at each iteration during the simulation. Hence the time step of the equivalent model should be set to the quotient of simulation time step divided by the iteration number. The time step of the output time series of PSS/E is equal to the simulation time step, which in the study case is $0.1s$. And the time step of equivalent model should be $0.05s$.

Hence the data obtained from the output time series should be re-sampled at a higher rate. In System Identification Toolbox, re-sampling is done by applying an anti-aliasing FIR (Finite Impulse Response) filter to the data and changing the sampling rate of the signal by decimation or interpolation [109]. When the data is re-sampled at a higher rate, the re-sampled values occurring between measured samples do not represent measured information about the system.

By comparing the estimated spectrum of the re-sampled signal to the original spectrum in Figure 5.3, we can find that the spectrum of the power and the voltage have good agreement in amplitude with the original signal. This suggests that the energy density of the signal has been preserved. Whereas the spectrum of re-sampled frequency data has higher amplitude than the original data. This contribution is due to the noise added to the signal by the re-sampling process.

With the sampling rate increasing, there are also some high-frequency noises added to all the re-sampled signals, as we can see from Figure 5.3.



(a) Active power and frequency (b) Reactive power and voltage

Figure 5.3: Spectrum analysis

5.3.3 Model Structure Selection

After the signal is ready, a model will be selected for system identification. The structure of model plays a key role in model performance.

Physics Based Model vs. Input/Output Model

- Physics Based Model

  A Physics Based Model (PBM) describes the system based on its physical concepts. Hence it is easily understood and adopted. When the system consists of a single type of components with similar characteristics (e.g.: coherent group), PBM is quite suitable.

However, when there is more than one type of dynamic component, or the characteristics of the same type of component differs a lot, it would be difficult to describe the system using a simple PBM. If we use a group of models containing a few different types of model, the computational difficulty will increase. On the other hand, the increasing number of parameters will lead to difficulty in estimating parameters.

In addition, the measurement we could get is the measurements of the system rather than those of the loads and generators individually. It is impossible to identify each of their parameters.

- Input/Output Model

  An Input/Output Model (IOM) describes the external systems in the input/output characteristics rather the detailed physical structures inside them. On the other hand, the structure of the input/output model has nothing to do with the type of the components in the system. Hence it has the advantage of being more applicable and convenient.

  The following rules should be taken into consideration to create IOM:

  1. The model should be as simple as possible
  2. The model should reflect the nature of the system
  3. The estimation of the model parameter should be simple
  4. The model should be easily transferred for analysis and control of the power system

In general, when a system has a single type of component (e.g.: generators, motors), PBM should be selected. It describes generators by a similar nonlinear model and has clear physical concepts so that it could reflect the inner physical phenomenon. It could also be compatible with other components in the system. On the contrary, when the composition of the system is complex, it is better to use IOM, whose structure could be very simple.

In this project, as the aim is to derive a dynamic equivalent of distribution network with EG, whose composition could be complex or the updated information are unavailable, IOM would be a more proper model for dynamic equivalencing.

Linear Model vs. Non-linear Model

Some important components in the power system such as generators, excitation systems governors and loads have nonlinear characteristics. These components and their associated control systems include saturation and output limitations.

There is still no common method to analyse the identifiably of nonlinear models. When a model is not uniquely identifiable, the meaning of attempting to estimate its parameters is questionable.

If the power system works around the steady state operating point (e.g.: disturbance is small enough or the electrical distance from the disturbance to the generator in the system is significant), a linear model could be chosen. Linear models are easy to be analysed and require less computational effort. They can provide a useful insight into the dynamic behaviour of the power system.

This is not very restrictive in practice. In dynamic equivalencing, it is reasonable to consider linear models for components of an external system [54]. In [47], the responses of the linearised model of a distribution network with EG are verified with the original nonlinear model. Good agreement suggests that the dynamic elements of the distribution network can be represented without considerable error by linear models.

In this project, linear models have been used for system identification.

Auto-Regressive Exogenous Model vs. State-Space Model

The choice of an appropriate model structure is the most important for a successful identification application [107]. It is difficult to say which model is the best in general. Therefore model structures with different orders should be tested to search for acceptably accurate equivalents.

Auto-Regressive Exogenous (ARX) models and state-space models have efficient algorithms to handle many model structures [107]. Generally, ARX models work as well as state-space model. However, with the complexity of the system increasing, subspace state-space identification algorithms can provide better identification of the pole positions than the ARX model [108].

Another problem could be encountered with an ARX model is that the order of the numerator and denominator polynomials have to be chosen in advance. For a system with unknown physical property or complex structure, it will cause difficulties in defining the order. State-space models, however, only require determination of the state-space order.

In this project, state-space models have been selected for the reasons listed above. A dynamic equivalent in state-space form can be implemented into the simulation package PSS/E as a user-defined model for verification purposes.

5.3.4 Identification Algorithms Selection

A State-space model can be written in the innovations form, which describes noise (See Ljung [107] page 99). The discrete form of this state-space can be written as:

$$x(kT+T) = Ax(kT) + Bu(kT) + Ke(kT) \tag{5.3}$$

$$y(kT) = Cx(kT) + Du(kT) + e(kT) \tag{5.4}$$

Where

T is the sampling interval

$x(kT)$ is the current state at time instant kT

$u(kT)$ is the input at time instant kT

$y(kT)$ is the output at time instant kT

$Ke(kT)$ is the process noise

$e(kT)$ is the measurement noise, which is normally regarded as white noise.

There are two commonly used identification algorithms for estimating a state-space model which have been integrated into the Matlab system identification toolbox:

- Prediction-Error Minimisation (PEM)
- Numerical algorithms for Subspace state-space system Identification (N4SID)

PEM

PEM is an iterative prediction-error method based on minimisation of a criterion. The basic idea of the PEM method is to construct a predictor and compare its predictions with available data using some suitable measure [107]. This predictor is not just a simulation of the system but also takes measurement data into consideration, which will substantially reduce the prediction error.

PEM uses optimisation to minimise the cost function V_N, which is defined as follows for scalar outputs (see [107] page 199):

$$V_N(\theta) = \sum_{k=1}^{N} \epsilon(kT)^2 \tag{5.5}$$

Where

N is the number of data samples.

$\epsilon(kT)$ is the prediction error, which refers to the difference between the measured output y and the predicted output $\hat{y}$ at time kT respectively.

$$\epsilon(kT) = y(kT) - \hat{y}(kT) \tag{5.6}$$

The subscript N indicates that V_N is a function of N and becomes more accurate for larger values of N.

N4SID

N4SID (see Appendix A) is an algorithm proposed by Van Overschee [110], which estimates a state-space model using a subspace method. The subspace method is a 'one shot' identification method rather than iterative. It relies on linear algebra and does not involve optimisation criteria.

A state variable plays the following roles in a state-space model:

1. The state is a function of the past input/output data
2. The state summarises all the information that is contained in the past input/output measurements. This is relevant for the prediction of the future output.

In this sense, the state is the interface between the past and the future, which is the fundamental fact for all subspace algorithms.

The subspace algorithms split the available input-output data into two blocks, which can be considered as the past and the future. The basic idea is to split the model into two subsystems: a deterministic subsystem, which is defined by the evolution of the system dynamics, and a subsystem, which purely driven by noise. The identification becomes a de-noising process. This is feasible since the system is assumed to be linear and hence works on the superposition principle.

The N4SID algorithm is easy to use and can generally give good results. It is always convergent (non-iterative) and numerically stable. However, it might perform poorly under certain situations when the system has a lack of excitation, which means the disturbance has not exchanged sufficient energy with the system [111].

Summary

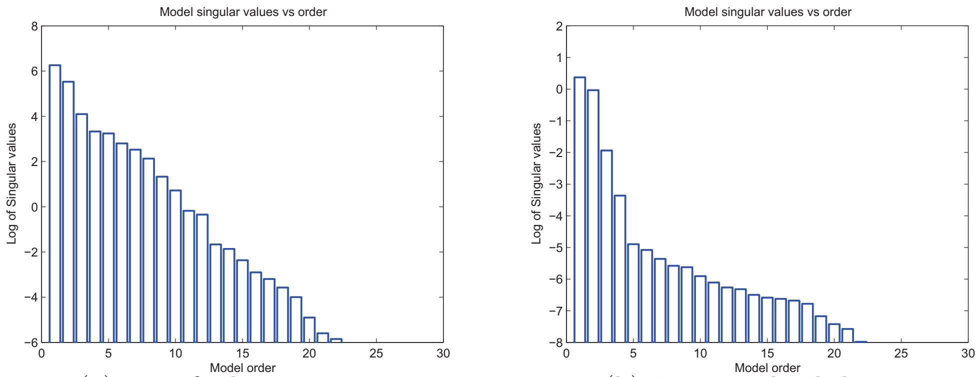
In general, the N4SID algorithm is fast and reliable but has less accuracy than the computationally heavier PEM algorithm. However, such differences in accuracy can be reduced when the strength of the signal is increased.

In this project, both PEM and N4SID algorithms have been used to derive discrete state-space models. These models are used to replace the distribution network with EG. Their performance in simulating dynamic responses of the original system are verified and compared in a simulation program.

5.3.5 Model Order Selection

The rank of the Hankel matrix is known to be the order of the system. The singular values of the Hankel matrix of the impulse response for different orders are graphed. An ideal singular value diagram can indicate the true order of the system by a significant drop in the singular values because the singular values only associated with noise should be smaller than those also associated with the system dynamics.

However under many situations, such behaviour is not apparent. This suggests that the modes in the system are either not excited sufficiently (which means not enough power is exchanged with the system modes) [108] or noise-contaminated (e.g. real data rather than simulated data) [112].



(a) Line fault (b) Reactive load disconnection

Figure 5.4: Singular value diagram for model order selection

Figure 5.4 shows the singular value diagrams of the signals collected from the simulations under different disturbances. The disturbance for Figure 5.4 (a) is a three phase line fault in the transmission network close to the distribution network. The disturbance for Figure 5.4 (b) is a small reactive load ($2MVar$) disconnection at the connecting bus between the transmission and the distribution networks.

In Figure 5.4 (b) a significant drop can be seen between the 4th and the 5th columns. As mentioned earlier, this drop can indicate the true order of the system, which is $4th$ in this case. Whereas in Figure 5.4 (a), the drop is not as obvious. Is it the case that noise contaminates the signals differently? As the signals are from simulations this appears unlikely. Did the small reactive load disconnection excite the system more than the fault? One possible explanation for this might be that disturbances like a fault could excite more dynamic modes of the transmission network than a load disconnection disturbance could do. Hence the signal involves a more dynamic behaviour of the transmission network. This problem will be discussed further in Section 6.2.4.

Disturbances like load disconnection tend to indicate the order of the distribution network easily. However, further simulations suggest that it is not a proper signal to be used for system identification in this project. The lack of richness of the signal leads to less accuracy of the model, especially when the model is working under other types of large disturbances.

As a result, a $4th$ order model is suggested to be able to represent the system. In [113], it is suggested that $4th$ order models would result in better estimates of modes. Lim [112] agrees that a reduced order model obtained by retaining only significant singular values might be poor in accuracy. In [107], it shows that as long as the basic properties of the system behaviour could be picked up by a model, it is unnecessary to tune the orders just to improve the fit. Hence state-space models over the order range from $4th$ to $6th$ have been used for system identification and compared for performance.

5.3.6 Dynamic Equivalent Model

The state space model derived using System Identification Toolbox of Matlab is represented in the following form:

$$x(t + Ts) = Ax(t) + Bu(t) \tag{5.7}$$

$$y(t) = Cx(t) + Du(t) \tag{5.8}$$

where

T is the sampling interval

$x(kT)$ is the current state at time instant kT

$u(kT)$ is the input at time instant kT

$y(kT)$ is the output at time instant kT

The coefficients of the model are in matrices. For example:

A =

	x1	x2	x3	x4	x5
x1	0.98705	-0.011057	-0.011249	-0.0029144	0.0057223
x2	-0.0169	0.98966	-0.0039157	-0.036558	-0.010731
x3	-0.25402	-0.063949	1.0466	0.2293	-0.20354
x4	0.087703	0.002135	-0.18908	0.30655	0.18851
x5	-0.55421	-0.09808	0.22954	0.80555	0.46343

B =

	V	f
x1	-0.94518	-82.864
x2	-2.1867	-135.4
x3	85.381	8563.2
x4	-137.44	-12818
x5	245.19	24109

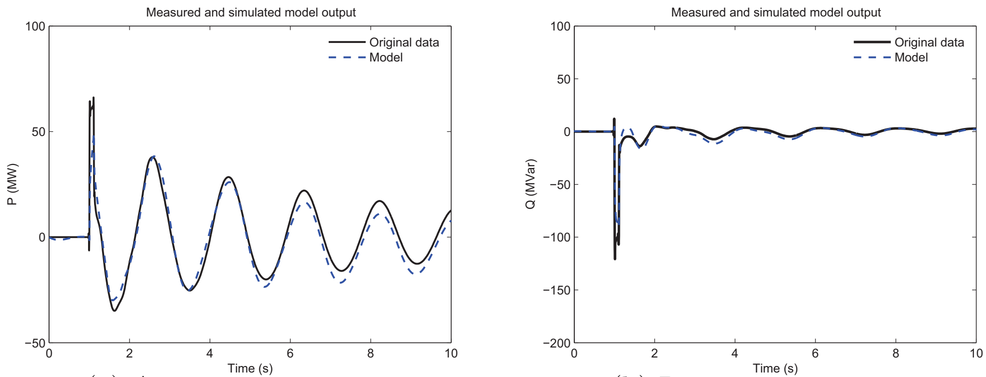
C =

	x1	x2	x3	x4	x5
P	-289.04	167.28	-1.4629	-3.2845	-1.5786
Q	-30.518	-232.24	4.5873	4.6307	0.82434

$$D = \begin{array}{ccc} & V & f \\ P & 0 & 0 \\ Q & 0 & 0 \end{array}$$

5.3.7 Model Outputs Analysis

After a model has been derived from the original data, the simulated output of the model corresponding to the original input should be compared with the original output of the system (the original data set or part of it).



(a) Active power (b) Reactive power

Figure 5.5: Comparison of original data and simulated model output

Figure 5.5 shows the simulated outputs of a $4th$ order state-space model identified by the N4SID algorithm using V, f as inputs and P, Q as outputs. The reasonable agreement between the simulated output and the original data may suggest that the problem is not very difficult and a relatively simple linear model is good enough.

The derived state space model with good performance in model outputs analysis is required to be implemented into a power system simulation program to replace the original external system as a dynamic equivalent. This dynamic equivalent is intended to be implemented in a way that interacts with the retained transmission system at each time step to give the simulated behaviour of the original distribution network. Verification should be done before it can be used for dynamic analysis and stability studies.

5.4 Model Verification

The derived dynamic equivalent model with good performance in fitting the simulations is then implemented as a PSS/E user-defined model to replace the full distribution network model in the original power system model (Figure 5.1(c)).

Verification is done by comparing responses subject to transient disturbances (e.g.: short-circuits) of the original distribution network model in Figure 5.1(b) and the dynamic equivalent model in Figure 5.1(c). The responses of a constant power equivalent model (see Figure 5.1(d)) are also given to indicate the beneficial effect of using dynamic equivalents.

5.5 Summary

In this chapter, the dynamic equivalencing methodology together with its key step, system identification, have been introduced. The data processing and model selections for deriving a proper dynamic equivalent model of a distribution network using the system identification approach has been considered. A state-space model is suggested to be selected and two popular identification algorithms for the state-space model are explained, the performances of which will be shown later. The whole system identification process has been done in Matlab system identification toolbox with data obtained from the dynamic simulation program PSS/E.

The obtained state-space dynamic equivalent model requires to be implemented into a simulation program for verification and application. The implementation procedure of the model will be described in the following chapter.

Chapter 6

Case Study on Scottish Power System

6.1 Introduction

A case study has been done on the Scottish Power system, which consists of the transmission network in southern Scotland and a distribution network located in Dumfries and Galloway, together with some small generators embedded in it.

Dynamic equivalent models have been derived and verified based on the dynamic simulation results of this power system. In terms of dynamic equivalencing, the study system is the transmission network and the external system is the distribution network.

6.2 Challenges and Solutions

6.2.1 Structure of Distribution Network

Most distribution networks in the UK are radial networks, or where interconnected are operated in the same way as radial systems by using normally open connection points. However, the conventional dynamic equivalencing approaches were mainly developed for transmission systems. Coherency-based approaches, for example, based on the fact that a group of machines swing coherently against generators in another area, are not suitable for dynamic equivalencing of a radial distribution network.

6.2.2 Embedded Renewable Components

Some renewable generators are based on induction machines, which have no synchronising torques. Also, some renewable generators are connected to the grid through electronic interfaces. This might separate the dynamics of the generator from the network. For the situations above, the coherency concept becomes meaningless. Moreover, the renewable generators in a distribution network may use various techniques and very different control systems, which makes them difficult to be aggregated [47]. Therefore, the coherency identification and aggregation based dynamic equivalencing may not be suitable in this case.

6.2.3 Information Availability

As mentioned in Section 2.4.4, the development of renewable generation means more small generators will be embedded into distribution networks. The technical information of these small generators are not always available to the transmission system operator. Furthermore, studies on validated models of some renewable components are not sufficient. Their dynamic effects are usually neglected by the transmission system operator although they may start to influence the dynamics of the transmission network due to the increasing penetration level.

Hence dynamic equivalencing based on the premise that the detailed model of the original power system is available is not suitable in this case. To solve this problem, a dynamic equivalencing methodology based on simulations and intended ultimately for measurements of power system responses has been developed in this project.

6.2.4 Transmission Network Effects

Introduction

The objective system that we want to derive the equivalent of, is the distribution network. In reality, the distribution network is always connected to the transmission network to distribute electricity from substations to customers. It is generally forbidden to disconnect the distribution network from the transmission network for testing purposes.

As a result, for dynamic equivalencing based on responses of the power system, the dynamic equivalent we could derive from this combined response is actually the dynamic equivalent of the whole power system, instead of the dynamic equivalent of the distribution network only.

Solutions

The challenge of how to reduce the transmission network effects on the system response and how to isolate the response of the transmission system from the response of the distribution system for dynamic equivalencing are discussed here.

The response of the power system subject to a disturbance is dependent on the location and the type of this disturbance. Hence a key point is to find out the disturbances which have minor effects on the transmission network so that the dynamic response of the system is mainly the response of the distribution network.

Based on the location of the disturbances, they could be grouped into two categories:

1. Disturbances in the distribution network

   Disturbances in the distribution network may have small effects on the transmission network. But they could change the topology of the system during the fault, which will more or less affect the accuracy of the equivalents.

2. Disturbances in the transmission network

   If we choose disturbances in the transmission network, the generator in the transmission network can also be affected. A model derived using these measurements is the equivalent of the full system rather than the distribution network only.

The solution for this could be to consider disturbances in the transmission network that are very close to the connecting bus between the distribution network and the transmission network.

Besides the location, the type of the disturbance is also important for disturbance selection. It should be noted that the disturbances used to derive equivalents should be practical (e.g.: line fault or tripping, load rejection etc.). Some types of faults can be easily tested by running simulations on computer but cannot be applied to a real power system.

Selection of the disturbances determines the quality and usability of the response data that we could obtain from the system, thus considerably affects the accuracy of the derived dynamic equivalent.

6.3 Implementation of Scottish Power System in PSS/E

6.3.1 Introduction

System identification techniques are typically applied to actual field measurements. Since such data was not readily available to ensure the variety of test cases required to study the proposed identification technique, data generated through time domain simulations of models of the test systems was used instead. Among a number of possible tools, the simulation tool Power System Simulator for Engineering (PSS/E) was chosen.

A PSS/E model of the Scottish power system for dynamic simulations has been implemented based on the power flow network data provided by the electricity company Scottish Power. Figure 6.1 shows a geographic overview of the transmission system, its major generators and the location of the distribution network that will be equivalenced.

The generators in the Scottish transmission network are listed in Table 6.1. Due to the lack of actual parameter information of the synchronous machines in the power system, typical machine parameters from textbooks (e.g.: Kundur's book [8]) have been used for the big generators. Machine parameters for smaller generators are based on some other reference material.

Bus Name	Generator ID	Capacity (MW)	Bus Name	Generator ID	Capacity (MW)
CAFA5-	1	5.0	CAFA5-	2	5.0
CHAP01	1	24.5	CHAP02	2	24.5
CHAP03	3	24.5	CHAP04	4	24.5
CHAP05	5	24.5	CHAP06	6	24.5
CHAP07	7	24.5	CHAP08	8	24.5
COCK01	1	230.3	COCK02	2	230.3
COCK03	3	230.3	COCK2-	E1	200.0
COYT1T	E1	200.0	EAST5-	1	5.5
EAST5-	2	5.5	GALA1Q	E1	30.9
GLLE5-	1	10.0	GLLE5-	2	10.0
KEOO5-	1	11.0	KEOO5-	2	11.0
KILS4-	E1	724.3	CHAP3-	M1	14.0
STHA2-	E1	412.9	STHA4-	E1	182.1
TONG5-	1	9.3	TONG5-	2	9.3
TONG5-	3	9.3	TORN4-	E1	200.0
TORN01	1	566.5	TORN02	2	566.5
HARK21	E1	200.0	HARK22	E1	200.0
HARK40	E1	200.0	STEW20	E1	200.0

Table 6.1: Generators in the Scottish transmission network

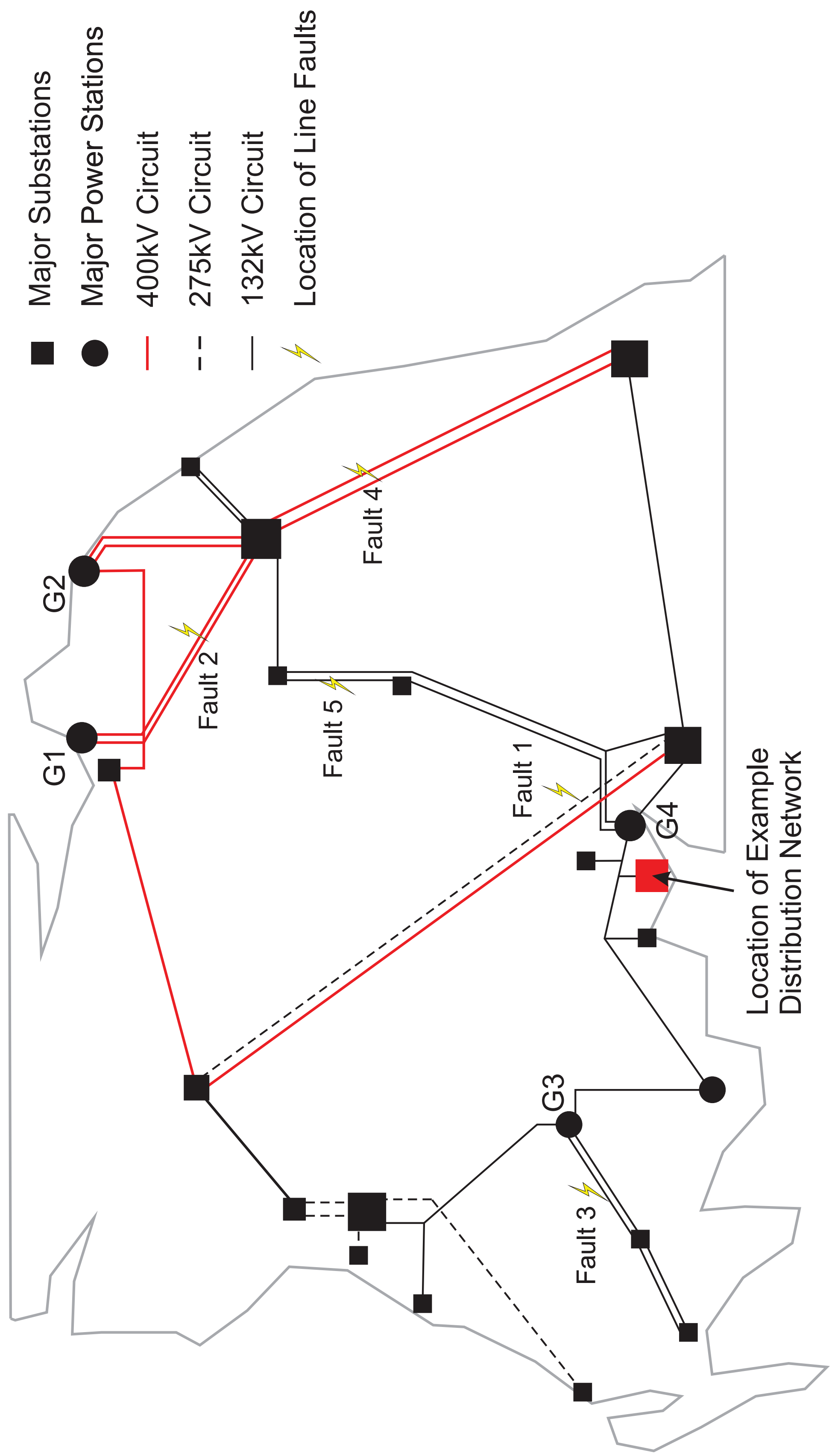


Figure 6.1: Map of Scottish Power System

Figure 6.2 shows the distribution network in Dumfries and Galloway (D&G). The D&G network information is accurate for transformer parameters and line parameters. For the purpose of studying a distribution network with a high penetration of embedded generators, some small fictitious generators (listed in Table 6.2) are implemented in this network model. The parameters of these fictitious embedded generators have been assigned with reference to typical generator parameters.

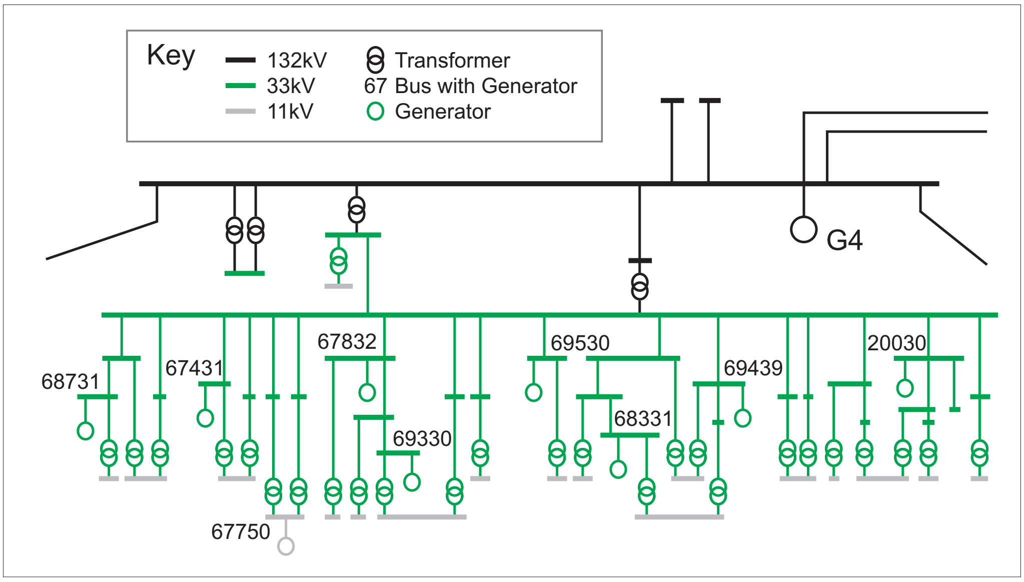


Figure 6.2: Dumfries and Galloway distribution network with embedded generators

Bus Number	Bus Name	Generator Type	Capacity (MW)	Voltage (kV)
67431	CARG3R	Diesel	2	33
67750	HEHA5-	Hydro	11	11
67832	LOCH3Q	Diesel	4	33
68731	STRD3R	Steam	25	33
69330	PENP3-	Steam	25	33
69439	LOCK3I	Diesel	2	33
20030	CANO3-	Diesel	4	33
69530	CATN3-	Steam	25	33
68331	MOFF3R	Steam	25	33

Table 6.2: Embedded small synchronous generators in D&G network

6.3.2 PSS/E Package

Introduction

PSS/E is a commercial software package for simulating, analysing, and optimising power system performance. It uses the Newton-Raphson (AC and DC) technique for regular power flow. Linear programming for optimisation in performing power flow studies, unbalanced fault analysis, and dynamic simulation are available to the user [114]. For this project, PSS/E is used to simulate disturbances (the fault event) during which the post fault operation is observed.

Strengths of PSS/E for this Project

PSS/E has been used by utility companies for decades. Besides the accuracy of results, the efficiency of activities involved in data preparation and debugging have also improved.

PSS/E has the capability to create specialised graphics, forms and tabular reports, which is useful for analysis. PSS/E also has the capability to create single-line diagram from network model. This is very useful as otherwise the diagrams would have to be created manually, which is very time consuming. Some wind turbine manufacturers have been developing wind turbine models in collaboration with PSS/E, system operators and research organizations. The most important feature of PSS/E for this project is its user-defined model, which is an crucial part of the modelling work. This allows implementation of models defined by the user's code, which makes it possible for this project to test the dynamic equivalencing results in a power system model.

6.3.3 Models Used in the PSS/E Package

Synchronous Generator Model

PSS/E contains documented and verified models of synchronous generators used in conventional power plants, and also the excitation control, governors, and power system stabilisers (PSS) of different concepts.

The equations of rotor motion for PSS/E are:

$$\begin{aligned} T_J \frac{d\omega}{dt} &= T_m - T_e - D_e(\omega - 1) \\ \frac{d\delta}{dt} &= (\omega - 1)\omega_0 \end{aligned} \tag{6.1}$$

where:

T_J is inertia time constant, s

T_m is mechanical torque, pu

T_e is electromagnetic torque, pu

D_e is the damping coefficients, pu

ω is rotor angular speed

ω_0 is synchronous angular speed

The synchronous generators are represented by a sub-transient level model with two rotor circuits in each axis, The model also considers the machine saturation. GENSAL, a 5th-order salient pole generator rotor, has been used to represent hydro generating unit in the power system. GENROU, a 6th-order wound rotor generator model has been used to represent all other type of conventional generators.

In PSS/E, there are two description of generator models, the explicit method (which does not include a transformer) and an implicit method (which has an inner transformer). In this project, implicit generator models are used in dynamic simulations. In reality, utilities usually implement external networks as an implicit generator model because it can control not only the power but also the voltage at the bus bar.

The excitation system and the speed governor models used for generators are listed in Table 6.3. These models are described in [115] and the details will not be repeated here. Briefly IEEEG1 is a 1981 IEEE type one governor model, EXST1 is a 1981 IEEEE type ST1 excitation system model, IEEEST is a 1981 IEEEE power system stabiliser model, and HYGOV is a hydro turbine-governor model.

Generator model	Governor model	Excitation system model
GENROU	IEEEG1	EXST1
GENSAL	HYGOV	IEEEST

Table 6.3: PSS/E Synchronous machine model

Doubly-Fed Induction Generator

The Doubly-Fed Induction Generator, or DFIG, is a generator with a variable frequency current source which can feed into the machine rotor.

This gives variable speed operation above and below synchronous speed. It is often used in wind turbines to improve capacity and efficiency.

Induction generators need to absorb reactive power, in other words, they have lagging power factors. The Grid Codes require DFIG machines connected to the grid to operate at power factor over the range $0.95 p.f.$ lagging to $0.95 p.f.$ leading [116]. Hence for DFIG, reactive power compensation is not required.

Most embedded wind generators use power flow control rather than voltage control. For wind generators under power factor control, as the wind fluctuates, real power generation will fluctuate so the reactive power generation will also fluctuate to maintain a constant power factor. Fluctuating reactive power can cause changes in voltage [27]. Hence the power factor should be set close to unity to reduce voltage variations.

The PSS/E wind generator model Ge1500 has been used for implementing wind generators in the distribution network. Ge1500 is a variable speed DFIG generator with rated power of $1.5MW$. The technical information on this wind generator model is provided by the manufacturers to PTI.

Load Models

PSS/E also contains dynamic models for representation of consumption centres at fluctuating voltage and system frequency.

Activity CONL is used to convert the loads. For each load to be processed, CONL allow users to specify the manner in which constant MVA load is to be apportioned, by specifying the percentages of the constant current and constant admittance load characteristics.

6.3.4 Equivalent Model Implementation in PSS/E

Introduction

Once an equivalent model is derived, it can be transfered into a proper form and embedded as Fortran code in a user-defined model. This model will do the calculations based on the embedded equations as well as the current network conditions during the simulation.

Some issues associated with implementing an user-defined model in PSS/E will be discussed as follows.

Time Step of the Equivalent Model

As the equivalent model will be in discrete state-space form, the first thing needed is to define its time step. The default simulation time step in PSS/E is $0.1s$ and the smallest iteration number in each simulation step is 2 during the network solution.

Since a user defined model is called at each iteration during the simulation, the time step of the equivalent model should be set to the quotient of simulation time step divided by the iteration number (i.e. $0.1/2 = 0.05s$). This can be done by resampling the output data before it is used to derive the equivalent models.

However, the initial iteration number is usually changed to a larger number during and after a fault. This would lead to the difficulty in obtaining data sets for system identification and model implementation, because the time step of the output data sets used to derive the equivalent models is not fixed and the actual time step in simulations for the discrete equivalent model can be different from its defined time step.

This problem can be solved by setting the maximum iteration number to 2 in the PSS/E simulation parameter settings. A number of simulation results showed that such change on the original system model in this case does not really affect the dynamic response of the model to the disturbance.

Coordinated Call Model

The goal of the dynamic equivalent model is to adjust the output power to a desired value which fits the dynamic response of the original model under the same operating conditions. The model should be called during network solution to calculate current injections which are dependent on the bus voltage.

To satisfy this requirement, the equivalent model should be implemented as a coordinated call model in PSS/E, which contains not only the differential equation responsibility but also the current injection responsibility [115]. This means that the model will be called at:

- the primary entry point for state variable calculations;
- supplementary entry points for current injection calculations.

Machine-Related vs Load-Related

The user-defined model in PSS/E is based on the standard models in its library. Machine-related and load-related models might be the most commonly used. They are based on standard machine and load models respectively.

Both machine-related and load-related model can be called coordinatively, which involves current injection during the network balance. Here the load-related model is selected for the implementation of dynamic equivalents due to the reasons listed below:

- Flexibility in Conversion

  In PSS/E, the power flow model needs to be converted before it can be used for dynamic simulation purpose. By such conversion, a machine model will be represented as a voltage behind an apparent impedance handled by a Norton equivalent [115].

  A ZIP load model, before being converted, can choose to be converted in different ways (e.g.: constant power, constant current or constant impedance). It gives more flexibility for the model to represent the system without modifications. For a distribution system, if the composition of loads can be known or estimated, it also makes sense to convert the model in particular ways for more accurate results.

- Bi-directional Power Flow

  Although the distribution system is embedded with a high penetration of distributed generation, the power may not necessarily flow from the distribution side to the transmission side. The power generated by the distribution system may be more than the local loads or the other way round, especially when variable renewable generation is being considered. Hence the output power of the equivalent model can be either positive or negative. In this case, using a machine-based model is not suitable as a generator does not produce negative power.

- Convenience

  A machine-related model requires specification of the parameter inputs of the machine, whereas a load-related model only requires outputs of real power and reactive power (P and Q).

Model Writing

As mentioned earlier, the dynamic equivalent models are in a discrete state-space form and have been implemented into PSS/E using the Fortran language. This model executes the following tasks:

- Call the load model and assign the states and constant data

- Describe the mathematical model

  In section 5.3.4, we introduced the discrete innovations form of state-space description. The noise source e is assumed to be white noise with zero mean. When the model is used for simulation, e will be taken as zero. This will not give less accurate results for the response of the actual system [109] (see page 1-9).

  From Equation (5.3) and Equation (5.4) we can get the following state-space model structure:

$$x(kT+T) = Ax(kT) + Bu(kT) \tag{6.2}$$

$$y(kT) = Cx(kT) + Du(kT) \tag{6.3}$$

- Calculate the desired output power (P_{new}, Q_{new}) on the basis of the mathematical model and the model input (V).

- Calculate the present output power value of the standard load model (P_{old}, Q_{old})

$$Load = PQ + aV + bV^2 \tag{6.4}$$

- Calculate the incremental current of the model to the system on the basis of the difference between the desired output and the present output

An example of writing a PSS/E user-defined model is shown in Appendix D. Such codes are saved as *.flx files.

Model Compilation

The compilation of the user-defined model is required before it can be used for PSS/E simulation. The preparation for dynamic simulation by compilation contains the following steps:

- Write the model code in dynamic model input files.
- Run this file in PSS/E to generate two subroutine files CONEC and CONET (which handle state variables and the network current injection respectively) and one compile file.
- In Compaq Visual Fortran environment compile the *.flx file, which contains the information of the user-defined model. The subroutine files are compiled.
- Link the model code to PSS/E

This compilation has been done using Python code. The code is shown in Appendix C with batch compilation available.

6.3.5 Summary

This section describes how to implement the mathematical model into the commercial power system analysis tool PSS/E. The dynamic equivalent models are in discrete state-space form. They are implemented as user-defined models based on the standard load model in PSS/E. The coordinated call functions calculate the state variables and also the current injections during dynamic simulations.

Suggestions are given from experience with user-defined model writing and compilations. Any of the issues mentioned above, if not given enough consideration, may cause problems in model implementation and lead to very incorrect results.

Like other standard models in PSS/E, the outputs of the model can be obtained by selecting the proper output channels. To ensure the model operates properly during dynamic simulation, it is important that the maximum iteration number in PSS/E simulations should always be set to 2 for the reason discussed in 6.3.4.

6.4 Conclusion

The dynamic equivalencing methodology developed in this project has been presented in this chapter, including its implementation in PSS/E. While the methodology is intended for use with real measurements, by necessity simulation results from detailed simulations are used.

Chapter 7

Verification of Equivalencing Method

7.1 Introduction

The previous chapter described how to implement a user-defined model into PSS/E. Simulations have been run on a power system model with the original distribution network replaced by various equivalents. Different state-space dynamic equivalents of the distribution network were tested.

The performance of an equivalent model can be examined by visually inspecting its dynamic response in the time domain. Mathematical measures, however, can provide a more thorough basis for comparison. In this chapter, some measures are introduced to evaluate the agreement between the dynamic response of the equivalent models and that of the original distribution system model. These measures include Critical Clearing Time (CCT), Mean Square Error (MSE), cross-correlation peak value and the positions of the Prony modes.

The dynamic equivalent taken in this chapter is a 4th order state-space model identified using the N4SID algorithm:

$$x(t+Ts) = Ax(t) + Bu(t) \tag{7.1}$$

$$y(t) = Cx(t) \tag{7.2}$$

$$A = \begin{bmatrix} 0.99307 & -0.0076103 & -0.015259 & 0.0050114 \\ -0.020365 & 0.99093 & -0.0042669 & 0.041907 \\ -0.031179 & -0.0046652 & 0.94608 & 0.18838 \\ 0.26243 & 0.039733 & 0.091099 & 0.46495 \end{bmatrix} \tag{7.3}$$

$$B = \begin{bmatrix} -0.33286 & -0.20998 \\ -1.3631 & 2.6174 \\ -6.4277 & 10.056 \\ 17.731 & -34.662 \end{bmatrix} \tag{7.4}$$

$$C = \begin{bmatrix} -311.92 & 176.95 & 1.4851 & 3.8864 \\ -34.265 & -233.9 & 1.3537 & -5.4422 \end{bmatrix} \tag{7.5}$$

This dynamic equivalent model was derived based on simulations obtained from the model of the Scottish Power system (Figure 6.1) including a distribution network in Dumfries and Galloway (see Figure 6.2) embedded with fictitious small generators listed in Table 6.2.

The simulated time series were recorded at the connecting bus between the transmission and distribution network after a short circuit line fault close to the connecting bus, cleared at $0.1s$ after the fault.

As mentioned in Chapter 2, the conventional distribution network was designed to accept power from the transmission network and is usually regarded as a passive circuit supplying load. Also EG was usually considered as negative load at an early stage since the impact of EG on transient stability was small (see section 3.5.2). Hence the contrastive model for this dynamic equivalent is a constant power equivalent model. The comparison of the two is to check if a dynamic equivalent performs better than a simple static equivalent of a distribution network.

7.2 Dynamic Response in the Time Domain

The performance of the equivalent model can be analysed by studying its dynamic response in the time domain after a disturbance. Firstly, the same fault used to derive the dynamic equivalent has been applied to the system. This was a short circuit line fault close to the connecting bus at the location Fault 1 (Figure 6.1) which cleared in $0.1s$.

7.2.1 Response at the Connecting Bus

The response of the dynamic equivalent model and that of the constant power equivalent model were collected at the bus bar interfacing the transmission system model. They were compared with the original distribution network model response.

Figure 7.1 shows the performance of the dynamic equivalent and the constant power equivalent in fitting the real power, reactive power, voltage and frequency (denoted as P, Q, V and f) of the original distribution network at the connecting bus under Fault 1, the same fault as the one used to derive the dynamic equivalent model.

From this figure, we can see that due to the short distance of the line fault to the distribution system, it has caused large oscillations (see the thick blue solid lines) in the power measured at the connecting bus. The dynamic equivalent (thin red solid lines) was observed to broadly reproduce these oscillations, whereas the constant power equivalent (thin green dashed lines) did not follow the oscillation and kept very flat after the fault. The difference could also be seen in the voltage and frequency responses although

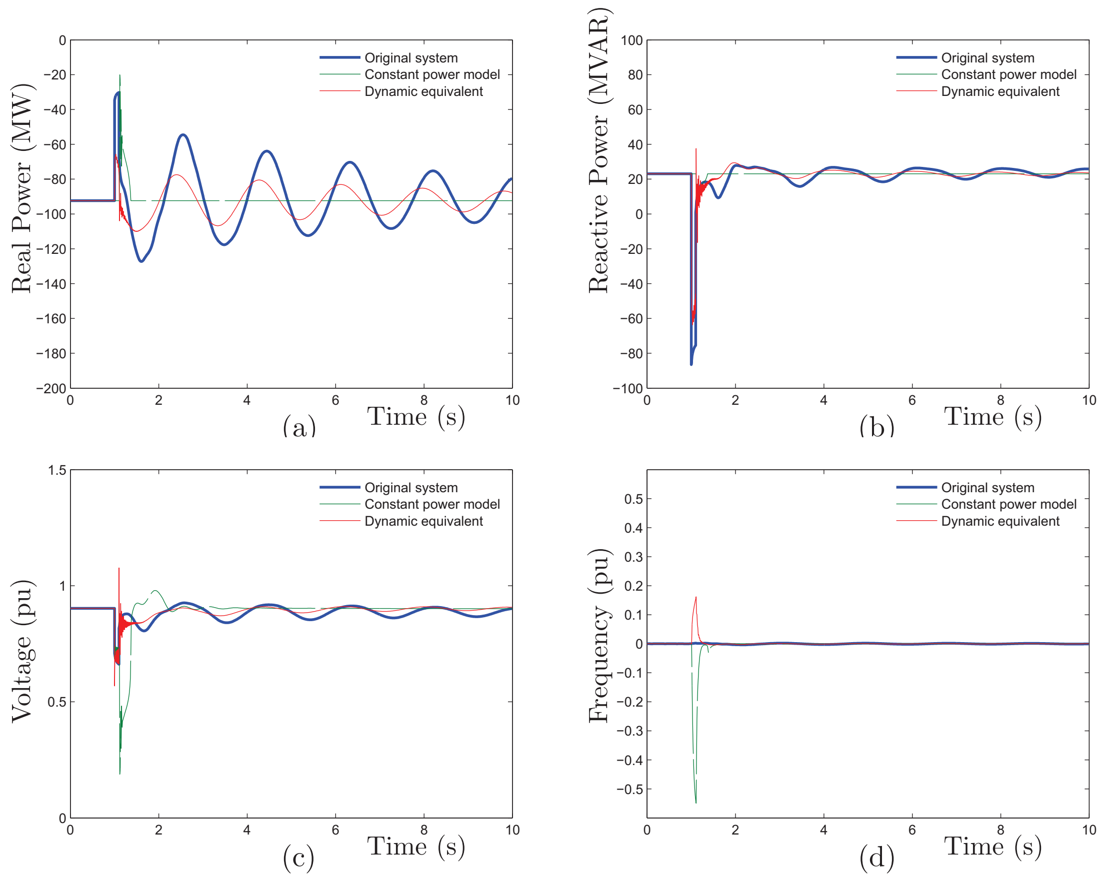


Figure 7.1: Dynamic response of the dynamic equivalent model and that of the constant power model at the connecting bus under a short circuit fault: (a) Real Power P; (b) Reactive Power Q; (c) Voltage V; (d) Frequency f

not as obviously since the oscillations are much smaller.

This suggests that the dynamic equivalent model has better performance than the constant power model in fitting the original system response.

7.2.2 Response in Machine Rotor Angle

In some literature on dynamic equivalencing, the performance of the dynamic equivalent was evaluated using only the disturbance used to derive the dynamic equivalent. For approaches like coherency-based dynamic equivalencing, the performance of the dynamic equivalent is dependent on the type and location of the disturbance used to identify the coherency of generators. Such dynamic equivalents may not work very well under other disturbances.

Hence the performance of the dynamic equivalent model under various faults has also been studied. Faults listed in Table 7.1 have been applied to the dynamic equivalent and the constant power equivalent model. Figure 6.1 shows the rough locations of these faults and some generators (G1 to G4) in the transmission network. The response of the equivalent models were compared with that of the original power system response. The responses are simulated rotor angles of the machines in the transmission network.

Number	Fault type	Clearing time
Fault 1	line fault	0.1s
Fault 2	line fault	0.1s
Fault 3	line fault	0.1s
Fault 4	line fault	0.1s
Fault 5	line fault	0.1s

Table 7.1: Faults applied

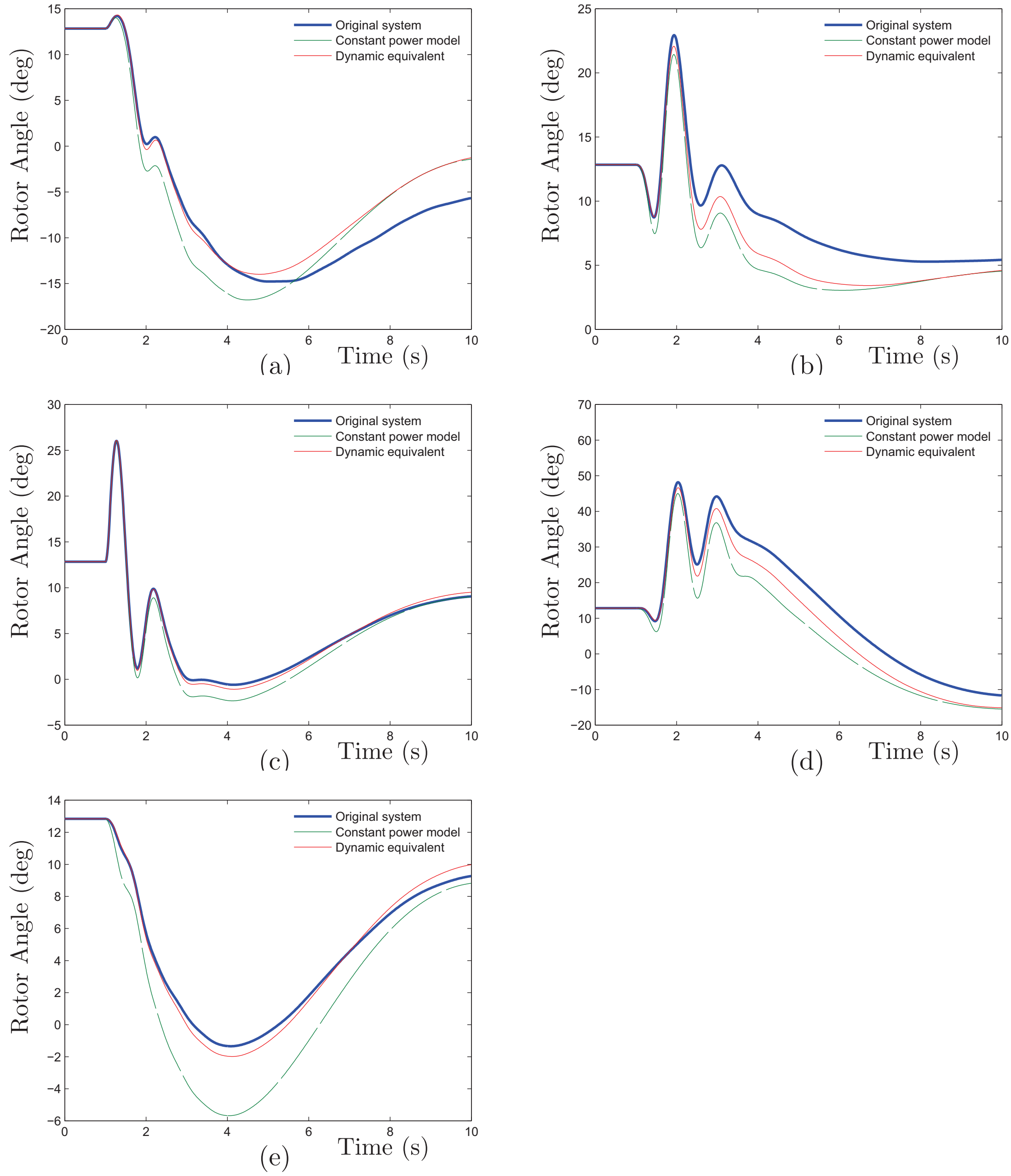


Figure 7.2: Dynamic responses of the dynamic equivalent and the constant power equivalent responses in rotor angle of G3 under various faults: (a) Fault 1; (b) Fault 2; (c) Fault 3; (d) Fault 4; (e) Fault 5

Figure 7.2 compares the time domain dynamic response of the dynamic equivalent and constant power models in simulating the rotor angle of machine G3. Graphs (a)-(e) show the dynamic equivalent (solid red lines), constant power equivalent (dashed green lines) and original responses (thick blue lines) under various faults. As we can see from the figure, the dynamic equivalent always has better agreement with the original responses, especially in the short time period after the faults.

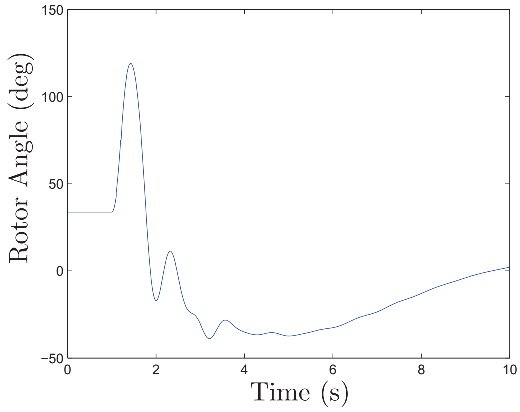
Simulation results showed that generally the dynamic equivalent model matches the original distribution network response better than the constant power model over the first $1-2s$ after the fault was cleared. However, after about $7-8s$ the constant power model seems to match the original system response as well as the dynamic equivalent does. In this case, we can still think that the dynamic equivalent works better than the constant power equivalent, as for power system dynamics and stability studies, the transient response immediately after the fault occurs is more important than the longer term response.

Conclusions can be drawn that for any fault tested in the simulation, the dynamic equivalent performs better than the constant power equivalent model in fitting original rotor angle responses of machine, especially over $2s$ after the fault was cleared.

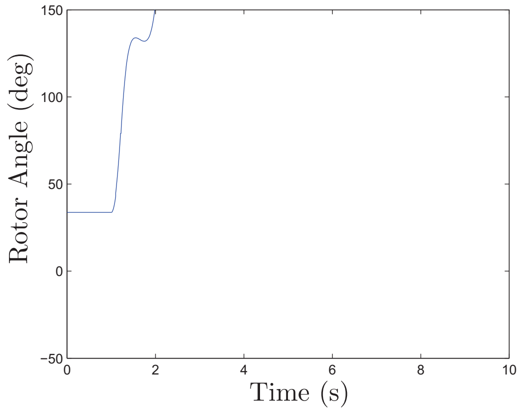
7.3 Critical Clearing Time

Critical clearing time, or CCT, is a commonly used measure to analyse the transient stability of power systems. It indicates the maximum time interval, within which the fault must be cleared for the power system to remain stable (i.e. to keep the synchronism of synchronous machines).

Figure 7.3 shows the rotor angle response of one synchronous machine under a line fault, which was cleared by tripping the line. On the left, the fault was



(a)



(b)

Figure 7.3: Influence of the clearing time on stability: (a) Clearing time = $0.21s$ = CCT; (b) Clearing time = $0.22s >$ CCT

cleared at $0.21s$ after the fault. The rotor angle achieved a new steady state after the oscillations. On the right, the fault was cleared at $0.22s$ after the fault. The rotor angle kept increasing and the machine lost synchronism, which indicates that the CCT of this machine for this fault is $0.21s$.

A reduced power system model based on an ideal dynamic equivalent should have a similar CCT as the original power system model for the same disturbance. Simulation on this power system model suggested that both the dynamic equivalent and the constant power equivalent have the same or very close CCT as the original distribution network under different fault conditions.

7.3.1 External Impedance Influence on CCT

For testing purposes, the impedance of the branch that connects the transmission network to the connecting bus interfacing to the distribution network, was increased step by step. This could increase the external impedance for the transmission network, and the increasing external impedance may have some impact on the CCT of the power system.

The CCTs of the power system based on the equivalent models after Fault 1 (see Table 7.1 and Figure 6.1) are listed in Table 7.2. R and X describes the branch impedance. P and Q are the real and reactive power flow from the connecting bus to the distribution system. The dynamic equivalent model tested is a 4th order state-space model derived using N4SID algorithm with Fault 1.

Model	R, X	P	Q	CCT		
		(MW)	(MVAR)	(a)	(b)	(c)
Distribution System	0.04 0.1	-32.601	39.2	0.21s	0.21s	0.21s
Distribution System	0.08 0.2	-32.604	43.014	0.22s	0.21s	0.21s
Distribution System	0.12 0.3	-32.401	37.192	0.22s	0.21s	0.21s
Distribution System	0.16 0.4	-32.381	32.152	0.22s	0.21s	0.21s
Distribution System	0.20 0.5	-32.375	28.269	0.22s	0.21s	0.22s
Distribution System	0.24 0.6	-32.296	25.021	0.22s	0.21s	0.22s

Table 7.2: Network effects of External Impedance: (a) Original distribution network model; (b) Constant power model; (c) Dynamic equivalent model

Table 7.2 shows the change in impacts of the external system on power system CCT with the external impedance increasing. (a) presents the CCT of the original power system with the full distribution network model; (b) presents the CCT of the transmission network with the constant power model; (c) presents the CCT of the transmission network with the dynamic equivalent model. Some conclusions can be drawn from this table:

- As we can see from the table, the CCT of the power system changed slightly from $0.21s$ to $0.22s$ with the external impedance changing. The CCT of the power system was not considerably changed by increasing the external impedance.
- In this case, a power system model based on the constant power equivalent model may have the same or a very close CCT to the original power system model. The difference between the original

power system model and the constant power based model is not significantly increased by increasing the external impedance.

- The dynamic equivalent has a CCT that is very close to the original power system model. However no obvious improvement in CCT can be seen when it is compared with the constant power equivalent.

7.3.2 Distribution System Penetration Influence on CCT

The impact of EG penetration level on the dynamics of a test system was investigated.

Increase Rated Power of Generator in the Distribution Network

Firstly, the penetration level is increased by increasing the rated power of the embedded generators. The rated real power of the generating units is listed in Table 7.3, with the locations of the generators shown in Figure 6.2.

Bus Number	Generation Type	Capacity (MW)				
		(a)	*(b)*	*(c)*	*(d)*	*(e)*
67431	Diesel	1.6	2	3	4	5
67750	Hydro	8.8	11	12	13	14
67832	Diesel	3.2	4	5	6	7
68731	Steam	20	25	26	27	28
69330	Steam	20	25	26	27	28
69439	Diesel	1.6	2	3	4	5
20030	Diesel	3.2	4	5	6	7
69530	Steam	20	25	26	27	28
68331	Steam	20	25	26	27	28

Table 7.3: Generators in distribution network

With the EG capacity listed in Table 7.3, the power flow from the transmission network to the distribution network (a-e) measured at connecting bus is listed in Table 7.4

	P (MW)	Q (MVar)	CCT
(a)	-9.7095	17.543	0.21s
(b)	-31.012	21.428	0.21s
(c)	-39.17	22.128	0.21s
(d)	-47.256	22.545	Generators in distribution system unstable at 0.21s
(e)	-55.277	22.414	Generators in distribution system unstable at 0.21s

Table 7.4: Increase Rated Power of Generator

Table 7.4 shows that:

- The CCT of generators in the transmission network is not affected by increasing the rated power of generators in the distribution network.

- Increasing the penetration with the fixed number of generators has more impact on the stability of generators in the distribution network than those in the transmission network. The CCT of the power system is changed due to the instability of generators in the distribution system when the generators in the transmission system keep their synchronism.

Decrease the Load in the Distribution Network

The penetration level was increased by decreasing the load in the distribution network. After adjusting the load in the distribution network, the power flow from the transmission network to the distribution network (f-g) measured at connecting bus is listed in Table 7.5

	P (MW)	Q (MVar)	CCT
(f)	-32.296	25.021	0.22s
(g)	-58.788	18.709	Generators in distribution system unstable at 0.19s

Table 7.5: Decrease the Load

This shows that:

- CCT is not considerably affected by decreasing the load of generators in distribution network.
- Decreasing the load of the distribution network may lead to the instability of generators in the distribution network.

7.3.3 Discussion and Conclusion

The above tests are applied to analyse the influence of the distribution network on power system stabilities using CCT as a measure. Factors like external impedance and penetration level have been adjusted to see how the distribution network influences the power system. Some conclusions can be obtained from the simulations.

- Increasing the external impedance does not considerably affect CCT of the power system.

- Increasing the penetration level with the fixed number of generators (either increasing the capacity of the generators or decreasing the load) may have more effect on the stability of generators in the distribution network than those in the transmission network. The CCT of power system is changed not because of the instability of generators in the transmission network but the instability of generators in the distribution network.

- Similar CCT results obtained from the three power system models suggest that CCT is not a good enough indicator to evaluate the performances of the equivalent model for a small distribution network.

- As the principal assumption for dynamic equivalencing, generators in the distribution system to be replaced by the equivalent should be kept stable.

7.4 Mean Square Error

7.4.1 Introduction

Mean square error, or MSE, is the average square of the difference between two signals. Under the same operating condition, the MSE value between the equivalent model response and the original model response can evaluate the ability of an equivalent in fitting the original model output. A good equivalent model should produce a small MSE value.

MSE between the equivalent model response and the original model response can be defined as:

$$MSE = \frac{1}{n}\sum_{i=1}^{n}(X_{equivalent}(i) - X_{original}(i))^2 \tag{7.6}$$

Where

X are variables (P, Q, V, etc.)

$X_{equivalent}$ and $X_{original}$ are data series of the equivalent model response and the original model response respectively

n is the data length of X.

7.4.2 Simulation Results

MSE of Response at Connecting Bus

For clearer comparison, the mean square error values between the equivalent model responses and the original model response were calculated and plotted in columns. Figure 7.4 displays the MSE values under Fault 1, the short circuit line fault used to derive the dynamic equivalent. The two graphs show the simulation results over $9s$ and $2s$ after the fault respectively. In each graph, the columns on the left represent the MSE values for the constant power equivalent and columns on the right represent those for the

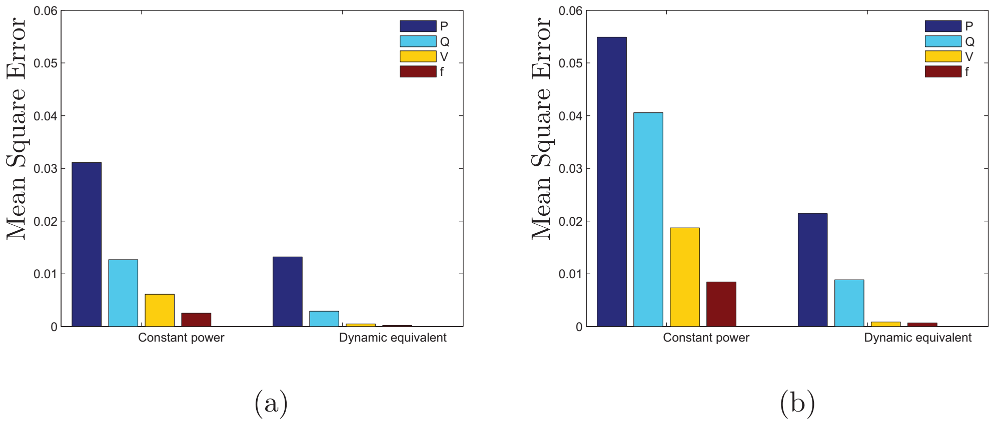


Figure 7.4: MSE in power, voltage and frequency responses of the dynamic equivalent model and the constant power model under a short circuit fault: (a) Over 9s after the fault; (b) Over 2s after the fault

dynamic equivalent.

Either over a short or longer time period after the fault, the small MSE suggests that the dynamic equivalent model can fit the responses in voltage and frequency at the connecting bus very well, whereas the performance of the constant power equivalent is obviously worse. The advantage of the dynamic equivalent can also be seen in the power response since it has much smaller MSE values than those of the constant power equivalent.

The calculation of MSE agrees with the conclusion that the dynamic equivalent model has overall better performance than the constant power model in fitting the responses. It also confirms the earlier conclusion that the dynamic equivalent performs better than constant power equivalent model in the first 2s after the fault.

MSE of Response in Machine Rotor Angle

Figure 7.5 shows the MSE values calculated from the response of the dynamic equivalent and those of the constant power equivalent for rotor angles of different machines in the transmission system. Besides Fault 1, which is used to derive the dynamic equivalent, some other short circuit line faults listed in Table 7.1 were also applied to the system for testing the

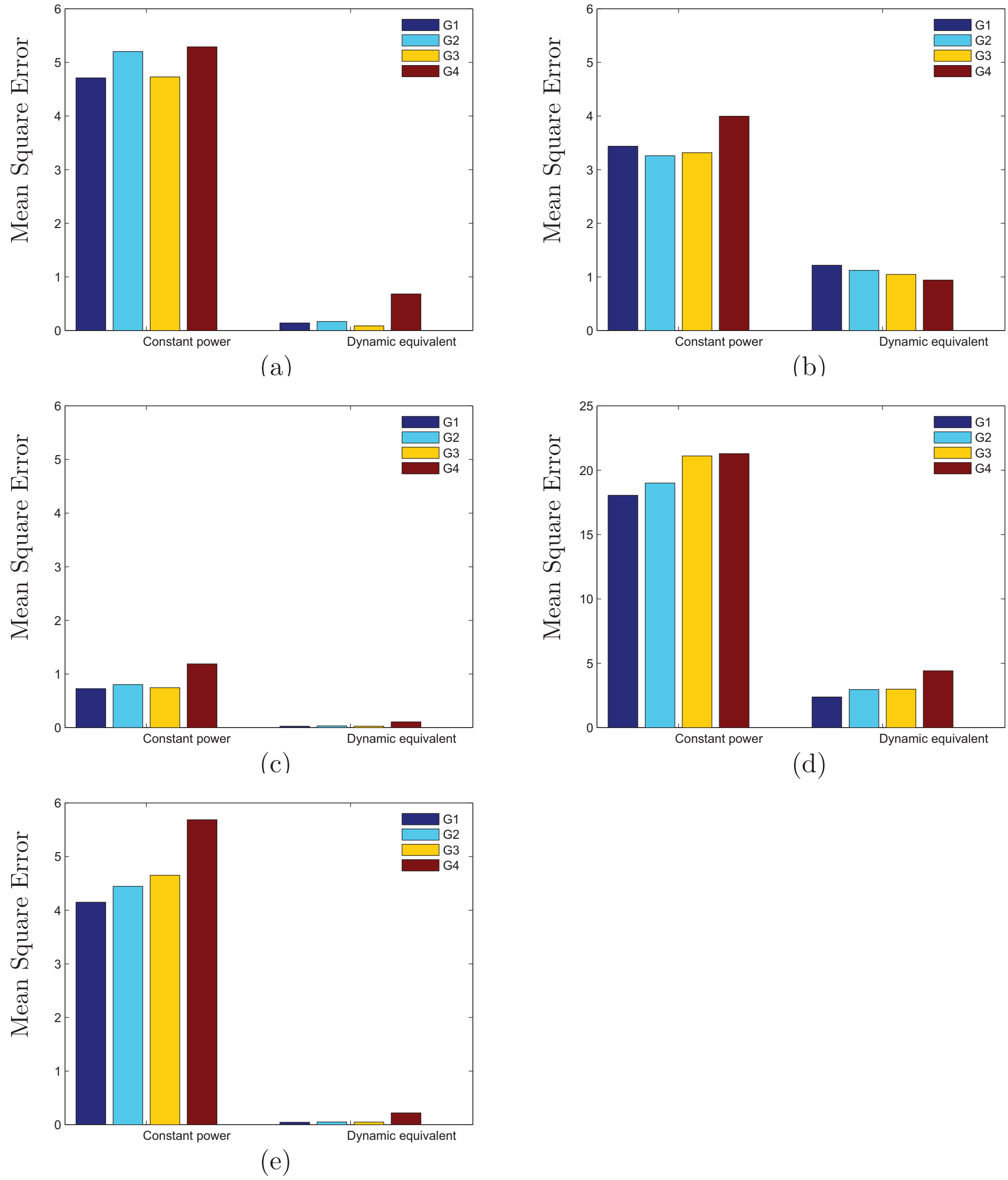


Figure 7.5: Comparison of the dynamic equivalent and the constant power equivalent in rotor angle MSE of different machines (G1-G4) under different faults: (a) Fault 1; (b) Fault 2; (c) Fault 3; (d) Fault 4; (e) Fault 5

equivalent responses in dynamic simulations. Graph (a)-(e) correspond to Fault 1-Fault 5 respectively. The MSE values were calculated based on the equivalent responses over $2s$ after the fault.

It is clear that the dynamic equivalent is overwhelmingly better than the constant power equivalent in capturing rotor angle response of any machines within the first $2s$ after the fault was cleared. The advantage of the dynamic equivalent can also be seen in their performances over $9s$ after the fault was cleared (See Appendix E). Moreover, both the dynamic equivalent and the constant power equivalent perform better in rotor angle response over the first $2s$ than they do over $9s$ after the fault.

7.4.3 Summary

MSEs between equivalent model responses and the original model response are calculated. These responses include P, Q, V, f values at the connecting bus and the rotor angles of machines in transmission network.

These MSE values have been used to evaluate the performances of equivalent models. Simulation results suggested that the $4th$ order state-space dynamic equivalent derived using N4SID algorithm have overall better performance than the constant power equivalent. The advantage of the dynamic equivalent is obvious especially in the first $2s$ after the fault has been cleared.

7.5 Cross-correlation Sequence

7.5.1 Introduction

Thinking of finding the similarity of signals, the term ”cross correlation” may come to mind. Cross-correlation sequence is a measure of similarity between two given signals, as a function of the relative time shift between two signals.

The complex cross-correlation of $x(m)$ with $y(m)$ is defined as:

$$R_{fg}(x) = f(x) * g^*(x) = \int_{-\infty}^{\infty} f(u+x)g^*(u)du$$

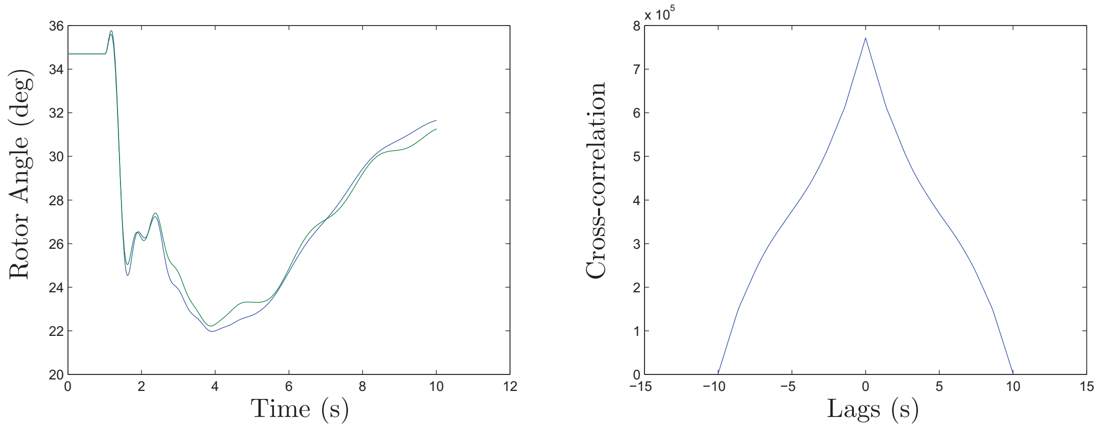
where

$*$ denotes convolution

$g^*(x)$ is the Complex conjugate of $g(x)$

The corresponding definition of cross correlation for sequences would be:

$$R_{fg}[m] = \sum_{n=-\infty}^{\infty} f[n+m]g^*[n] \tag{7.7}$$



(a) (b)

Figure 7.6: Cross-correlation: (a) Two signals in rotor angle; (b) Cross-correlation sequence of the two signals

Figure 7.6 shows an example cross-correlation sequence for two signals. These two signals (displayed in graph (a)) are correlated and their cross-correlation sequence (displayed in graph (b)) shows this relationship. In graph (b) we can see that around $time = 0$ there is a peak value of the sequence, which corresponds to the time shift between the two signals when they have the largest similarity. If this peak value was shifted from 0 in the x-axis, it would indicate a time lag between the two signals.

To evaluate the performance of the equivalent models, the cross-correlation sequences of the equivalent model and the original model response are calculated. The peak values of these cross-correlation sequences, which represent the maximum similarities of the equivalent responses to the original model responses, were used to evaluate the agreement of the two. A better equivalent model should have a larger cross-correlation sequence peak value.

The difference in MSE shown earlier is partly caused by a phase shift (i.e. lead or lag) in the response of the equivalent model from the original model. Two signals, which have similar amplitudes and waveforms, may have a large MSE due to phase shift. Cross-correlation sequence peak value can evaluate the model performance regardless of the inaccuracy caused by phase shift.

7.5.2 Simulation Results

Figure 7.7 compares the dynamic responses of the constant power model and the dynamic equivalent model under various faults. Graphs (a)-(e) show their performance under Fault 1 - Fault 5 over $2s$ after the fault was cleared. The cross-correlation sequence peak values of machine rotor angles were represented by columns. The right column group is for the constant power model and the left one is for the dynamic equivalent model.

As mentioned earlier, a larger cross-correlation peak value suggests a better agreement of signals regardless of phase shift. In Figure 7.7, under different faults from Fault 1 to Fault 5, for different machines from G1 to G4, the rotor angle response of the dynamic equivalent system always has a larger cross-correlation peak value than that of the constant power model system, which suggests the response of the dynamic equivalent system has better agreement to the original system than that of the constant power model

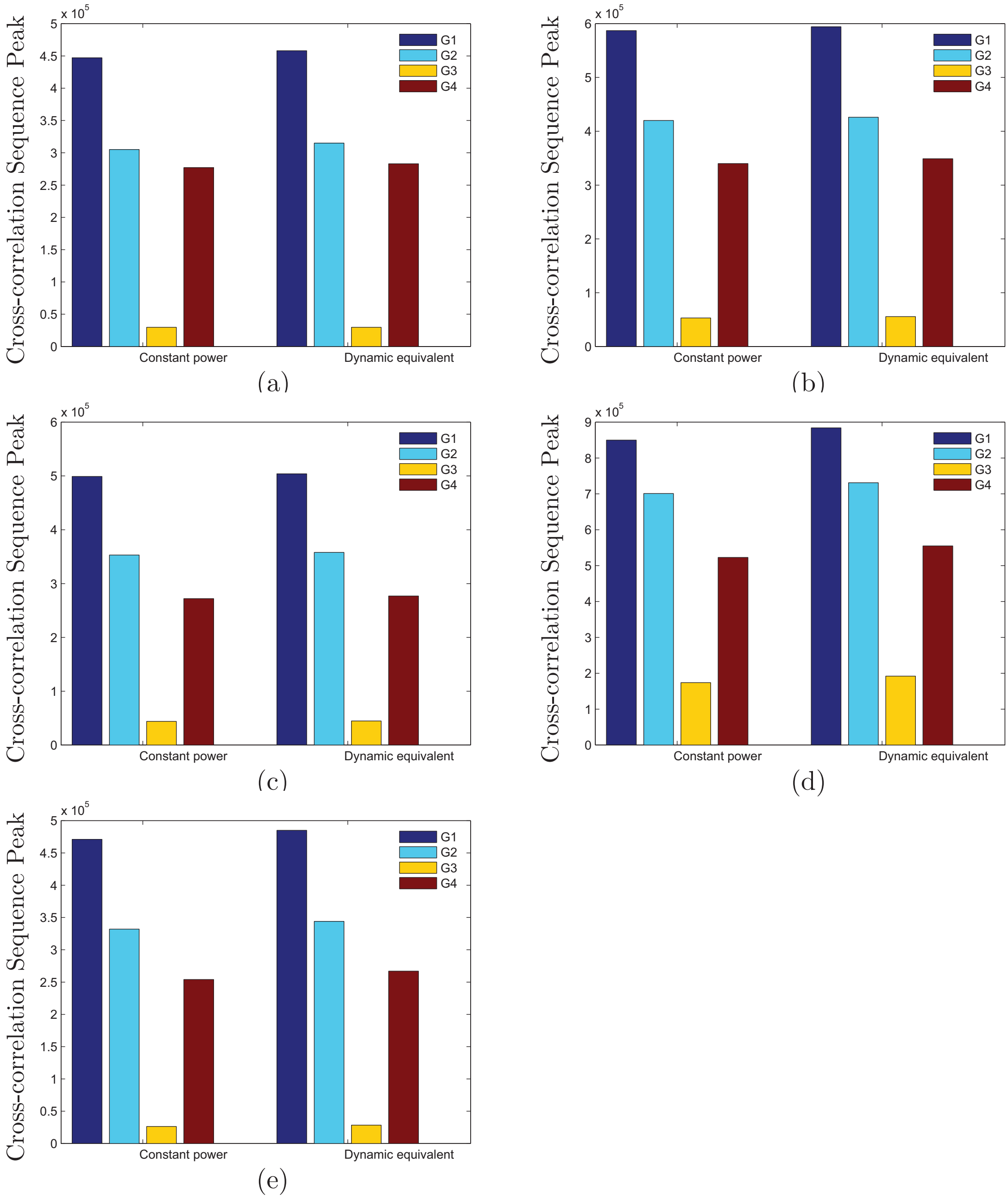


Figure 7.7: Comparison of the equivalent responses in rotor angles of different machines under different faults: (a) Fault 1; (b) Fault 2; (c) Fault 3; (d) Fault 4; (e) Fault 5

system. This means, taking no consideration of the inaccuracy of equivalent models caused by phase shift, the dynamic equivalent model still performs better than the constant power model.

7.6 Prony Modes

7.6.1 Introduction

Prony analysis can be used to identify the modes from the signals. For analysis, the Prony modes of the power system were identified from the dynamic response in rotor angles of machines at the transmission network. The Prony modes derived from the equivalent-based reduced power systems were compared with those derived from the original power system.

Modes of Power System

The real modes of the power system, as seen in Figure 7.8, can be divided into 4 groups according to their frequency levels. Each group of modes of the power system are represented by one or a few Prony modes. The 4 mode groups are:

- System modes

  System modes are non-oscillation modes. They are the most basic modes, which involve all the generators in the system. The close-to-zero system modes correspond to the angle state. And the real-negative system modes correspond to the speed state (system frequency). System modes are not of concern in this study.

- Inter-area modes

  Inter-area mode oscillations are associated with the generators in one area swinging coherently against the generators in the other area.

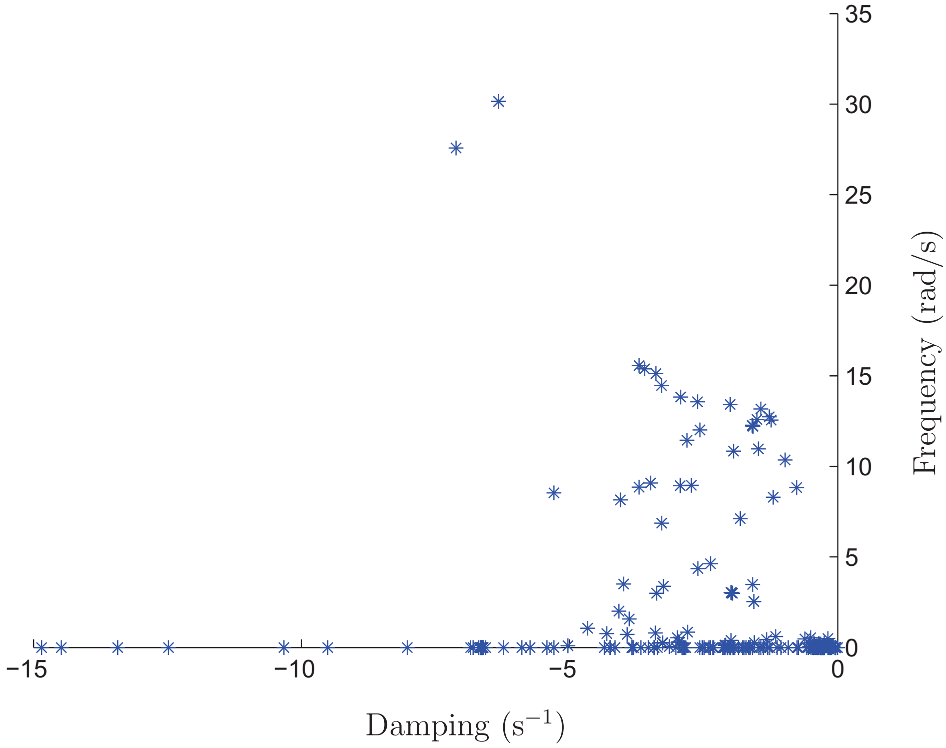


Figure 7.8: Real modes of the power system

These modes have lower frequencies and are within the range of 0.2 to $1Hz$ [117].

- Local modes

  Local mode oscillations are associated with single generators. Their frequencies are typically in a range of 0.5 to $2.5Hz$ [8].

- High frequency modes

  High frequency modes oscillations have even higher frequencies.

Prony Analysis

Prony analysis (described in Appendix B) can identify the damping and frequency of the modes from a signal.

It should be noted that the number of the modes needs to be determined in advance. Usually this number is less than the number of the real modes and is associated with the sample number of the data series used for identification. Hence each identified Prony mode may represent more than one real mode of the system. The number of the Prony modes selected for this project is 10, which is enough to reproduce the signal obtained.

The shift of the Prony modes and their corresponding residuals can be used as measures for evaluating the equivalent accuracy. A good reduced model should have similar modes and corresponding residuals to those of the original model.

7.6.2 Results

Machine $G1$

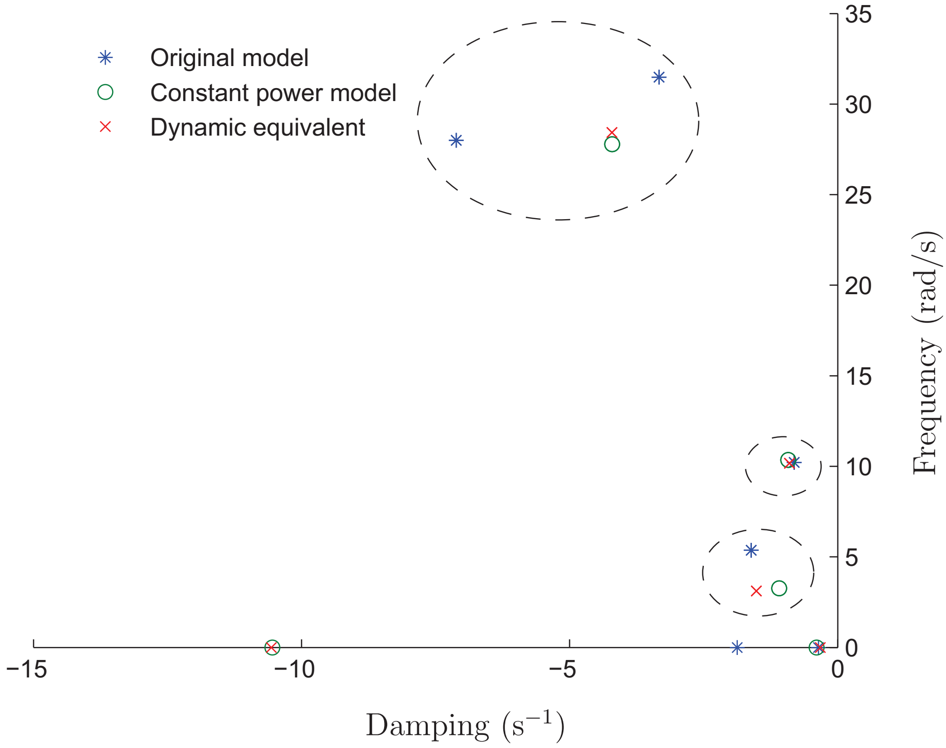


Figure 7.9: Prony modes identified from rotor angle responses of machine G1 under Fault 1

Generator $G1$ is located far from the equivalent network and the disturbance. The real power generated by $G1$ is $230.33MW$. Prony modes were identified by Prony analysis from the rotor angle response of $G1$ under Fault 1 and displayed in Figure 7.9. The Prony modes of the constant power equivalent system are represented by the green circles and those of dynamic equivalent are represented by a red cross. These Prony modes were compared with those for the original system, which are represented by the blue asterisks.

The first circle from the real axis marks the interarea modes, which are inferred as $Mode1$. In theory, the major difference between the constant power equivalent and the original system is usually the shift of inter-area mode. From Figure 7.9 we can find that the constant power equivalent has a shift in the inter-area mode positions. The figure suggests that the dynamic equivalent also has such a shift. But the difference is not as big as the one of constant power, especially in the damping of the mode.

The second circle from the real axis marks the machine modes, which are inferred as $Mode2$. Both the constant power equivalent and the dynamic equivalent reproduce this mode of the original system very well.

The third circle from the real axis marks the high frequency modes, which are inferred as $Mode3$. High frequency modes may shift largely on the s-plane. In Figure 7.9, two Prony high frequency modes of the original system were represented by one Prony mode of the equivalent.

Machine $G2$

Generator $G2$ is also located far from the equivalent network and the disturbance. $G2$ has real power output at $566.5MW$. Prony modes were identified by Prony analysis from the rotor angle response of $G2$ under Fault 1 and shown in Figure 7.10.

In Figure 7.10, one Prony mode of the original system represents both the machine modes and inter-area modes. Hence $Mode1$ and $Mode2$ circles

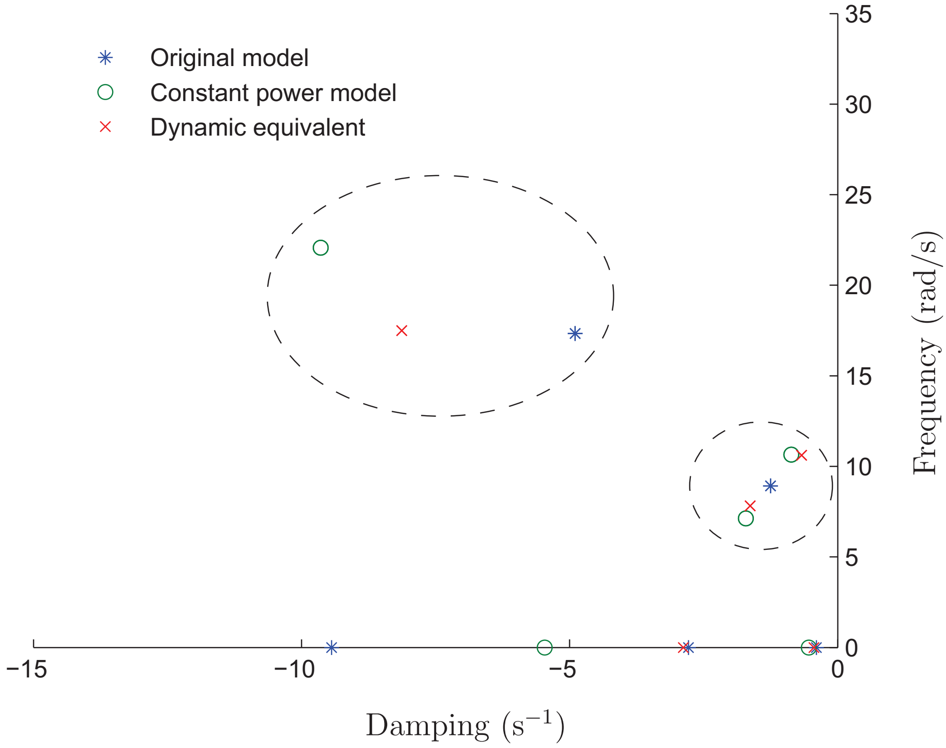


Figure 7.10: Prony modes identified from rotor angle responses of machine G2 under Fault 1

become one. $Mode1$ and $Mode2$ of the constant power equivalent has a similar performance to that of the dynamic equivalent.

The high frequency modes of the equivalent still has a large shift. The shift in high frequency modes of the dynamic equivalent is better than that of the constant power equivalent.

Machine $G3$

Machine $G3$ is a small generator with real power output at $11MW$. The modes identified by Prony analysis from the rotor angle response of $G3$ under Fault 1 are shown in Figure 7.11. $G3$ is neither close to the fault nor to the distribution network. Hence the mode positions of the constant power and most dynamic equivalent models are very similar to the original system.

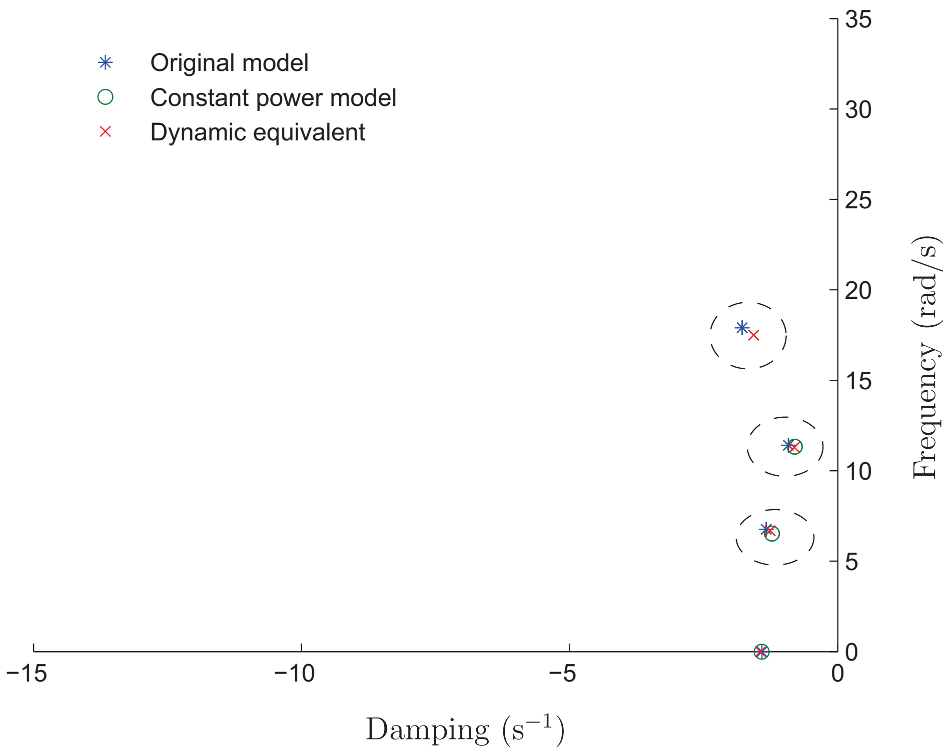


Figure 7.11: Prony modes identified from rotor angle responses of machine G3 under Fault 1

From Figure 7.11, it can be seen that both constant power and dynamic equivalents reproduced the inter-area and the machine modes of the original power system very well. These two modes almost have no shift from the modes of the original system.

The dynamic equivalent also tracked the high frequency mode of the original system and has only a small shift from it. The high frequency mode of the constant power equivalent, was however missing.

Machine $G4$

Generator $G4$ is quite close to the equivalent and the disturbance. It is a small generator with a real power output $24.5MW$. The modes identified by Prony analysis from the rotor angle response of $G4$ under Fault 1 are shown in Figure 7.12.

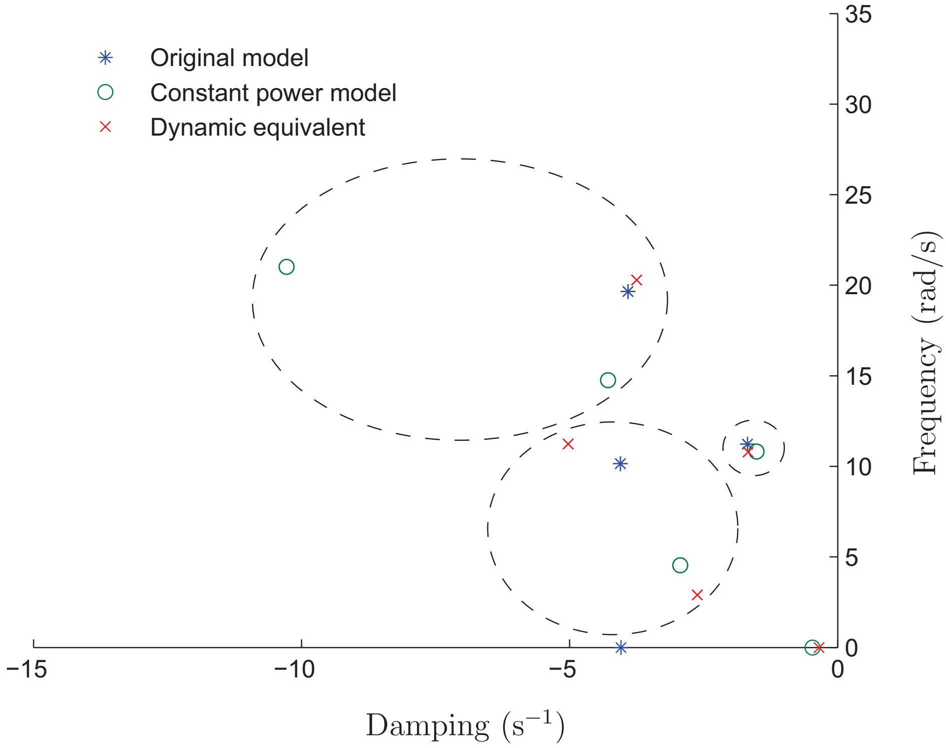


Figure 7.12: Prony modes identified from rotor angle responses of machine G4 under Fault 1

Both the equivalents performed less than ideally in representing the right mode positions. One inter-area mode of the dynamic equivalent is quite good but the other is far away from the mode of the original power system. This is because $G4$ is very close to both the distribution system and the fault. It is less reasonable to assume that representing the distribution system as a linear model can be good enough. The dynamic equivalent still has a slight advantage, especially in the reproducing the high frequency modes.

Conclusions

Prony analysis can be used to identify modes from the dynamic response of a system. It has been applied to the 4th order state space dynamic equivalent and the constant power equivalent reduced power system. The Prony modes of the two reduced power systems were compared with those of the original power system to evaluate their performance in representing the right positions of the original modes on the s-plane.

Rotor angle responses at different machines in the transmission system were used to identify the Prony modes. These machines locate in different distances from the distribution network (or distribution network equivalents) and have different generating capacities. The positions of Prony modes identified from any of these machine responses indicate better performance of the dynamic equivalent than the constant power equivalent.

7.6.3 Conclusions

This chapter compares the performance of a 4th order state-space form dynamic equivalent with that of a constant power equivalent as an alternative to the original distribution network model which has embedded small synchronous generating units.

Along with visually analysing the dynamic response of the system, some measures are used for evaluating the equivalent performance in fitting the original system response. These measures include mean square error, cross-correlation sequence peak values and Prony mode positions. All such measures suggest the better agreement of the dynamic equivalent responses to those of the original power system than the constant power equivalent.

Chapter 8

Factors Influencing Model Performance

8.1 Introduction

In the previous chapters, the system identification method has been used to derive a 4th-order state-space dynamic equivalent. The dynamic equivalent was compared with a constant power equivalent in simulating the responses of the original distribution network embedded with synchronous generators.

Some measures such as MSE, cross-correlation sequence and the positions of Prony modes, were introduced to evaluate the performance of the equivalent models. In this chapter, these measures will be used to evaluate the performance of different dynamic equivalents alongside the 4th-order N4SID state-space model.

Factors that affect the equivalent performance, such as the selection of the disturbances and the model structure applied in deriving dynamic equivalent models, will be discussed in detail.

The system identification dynamic equivalencing method has also been applied on the distribution network with embedded DFIG units. The application of DFIGs will be compared with the distribution network embedded with synchronous generators.

8.2 Double Input Model vs Single Input Model

8.2.1 Introduction

In this project, the dynamic equivalents in state-space form were derived by system identification using the data either with double inputs voltage V and frequency f or with single input V. In this section, we will focus on how the number of the inputs influence the performance of the dynamic equivalent model.

Model	Model description	Model input
Model 1	Constant Power model	
Model 2a	4^{th} order N4SID state-space model	V, f
Model 3a	5^{th} order N4SID state-space model	V, f
Model 4a	6^{th} order N4SID state-space model	V, f

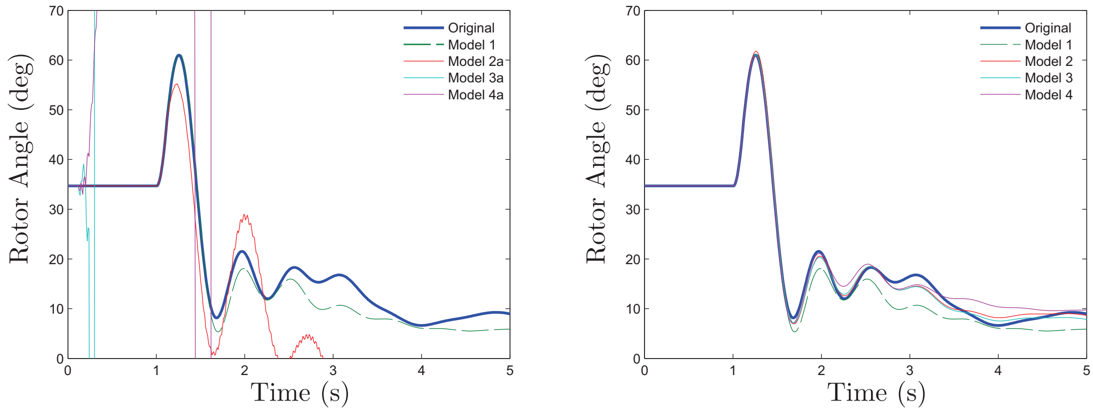
Table 8.1: Double-input Equivalent Models

Model	Model description	Model input
Model 1	Constant Power model	
Model 2	4^{th} order N4SID state-space model	V
Model 3	5^{th} order N4SID state-space model	V
Model 4	6^{th} order N4SID state-space model	V

Table 8.2: Single-input Equivalent Models

A list of state-space models shown in Table 8.1 were derived using double-input N4SID state-space models with orders from $4th$ to $6th$ (see section 5.3.5). The state-space models shown in Table 8.2 were derived using single-input N4SID state-space models with orders from $4th$ to $6th$.

8.2.2 Simulation Results



(a) Double-input models (b) Single-input models

Figure 8.1: Influence of the number of inputs on dynamic equivalent model performance: Model 1, Constant power model; Model 2a, 4^{th} order double input dynamic equivalent model; Model 3a, 5^{th} order double input dynamic equivalent model; Model 4a, 6^{th} order double input dynamic equivalent model; Model 2, 4^{th} order single input dynamic equivalent model; Model 3, 5^{th} order single input dynamic equivalent model; Model 4, 6^{th} order single input dynamic equivalent model.

In Figure 8.1, the left graph shows the dynamic behavior of a generator located very close to the distribution network under a line fault close to the connecting bus. The thick solid line and the thin dashed line are the original distribution network model and the constant power model (Model 1) respectively. The thin solid lines represent the responses of the $4th$, $5th$ and $6th$ order equivalent models derived from the data with two inputs V and f (Models 2a to 4a in graph(a)).

The performance of single-input state-space models are shown on the right graph of Figure 8.1, where the thin solid lines now represent the responses of the $4th$, $5th$ and $6th$ order equivalent models derived from the data with single input, voltage V (Models 2 to 4 in graph(b)).

8.2.3 Conclusions

The results suggest that the performance of the equivalent models derived from the two inputs, V and f, are worse than those derived from the V input only. The figure shows that, for a same order state-space model derived using the same identification algorithm, the double-input models show system instability when the original power system is stable. The single-input models, however, remain stable and provide good agreement with the original power system response.

The conceivable reason for this is because that frequency f at the connecting bus may change very insignificantly in the original system model. This lack of richness of the signal is not good for deriving an accurate equivalent model. Once such an double-input equivalent model is implemented, a small variation in f can lead to a considerable change in the model output, which will severely affect the model performance and hence cause instability in the power system.

In this case, the dynamic equivalent should be derived using single-input data instead of double-input data.

8.3 N4SID Model vs PEM Model

8.3.1 Introduction

For the single-input model, identifying the state-space model using different algorithms with different orders gives various type of models. These various model types can be applied to a same input-output data series to derive different dynamic equivalents. The type of the model can influence the performance of the dynamic equivalent, which was derived with it.

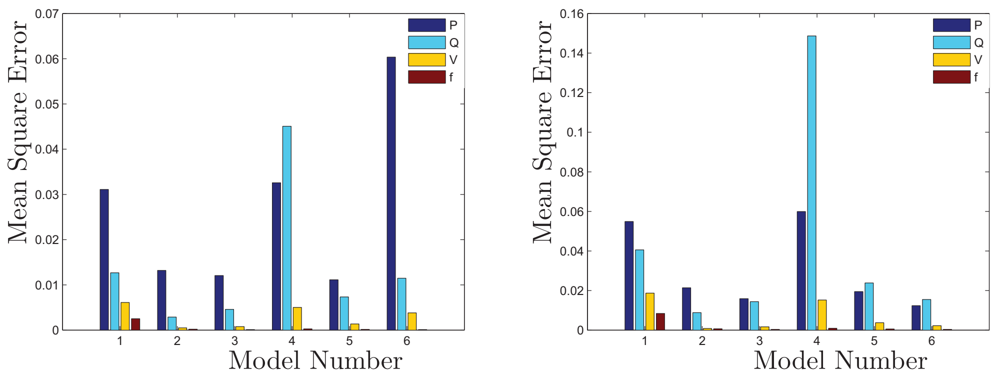
A list of state-space models shown in Table 8.3 were derived using two different algorithms, the N4SID subspace algorithm and the PEM algorithm (see section 5.3.4), with several model orders.

Model	Model description	Model input
Model 1	Constant Power	
Model 2	4^{th} order N4SID state-space	V
Model 3	5^{th} order N4SID state-space	V
Model 4	6^{th} order N4SID state-space	V
Model 5	4^{th} order PEM state-space	V
Model 6	5^{th} order PEM state-space	V

Table 8.3: Equivalent Models

8.3.2 Results and Conclusions

Under Fault used to Derive Dynamic Equivalent



(a) Over 9s after the fault (b) Over 2s after the fault

Figure 8.2: MSE in power, voltage and frequency responses of different equivalent models under a short circuit fault - Fault 1: Model 1, Constant power model; Model 2, 4^{th} order N4SID dynamic equivalent model; Model 3, 5^{th} order N4SID dynamic equivalent model; Model 4, 6^{th} order N4SID dynamic equivalent model; Model 5, 4^{th} order PEM dynamic equivalent model; Model 6, 5^{th} order PEM dynamic equivalent model.

Figure 8.2 shows the performance of different equivalents in fitting the real power, reactive power, voltage and frequency at the connecting bus (denoted as P, Q, V and f) under Fault 1 (short circuit line fault located close to the connecting bus and cleared at $0.1s$).

From this figure we can find out that:

- f: The brown columns suggested that, in ether $2s$ or $9s$ after the fault, all these dynamic equivalent models fit the response in bus frequency very well and perform better than the constant power model.

- V: The yellow columns suggested that, in either $2s$ or $9s$ after the fault, all these dynamic equivalent models have better performance in fitting the bus voltage than the constant power model.

  Meanwhile, the $4th$ (Model 2) and the $5th$ (Model 3) order N4SID state-space dynamic equivalent models have better performances than the $6th$ (Model 4) order one. The $4th$ (Model 5) order PEM state-space dynamic equivalent model has better performance than the $5th$ (Model 6) order one. The $6th$ order PEM model is not listed in Table 8.3 nor shown in this figure as it causes system instability after the faults and leads to huge MSE values.

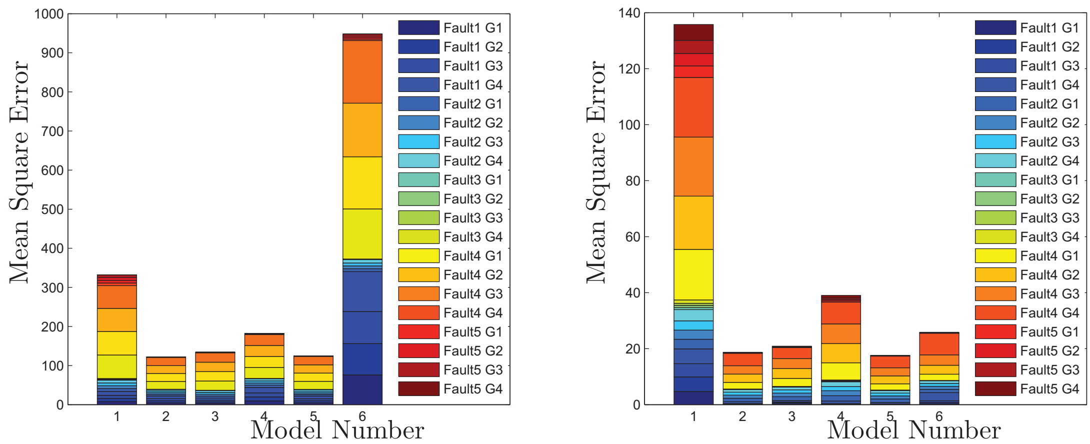
  A conclusion can be drawn that a lower order state-space dynamic equivalent may provide better performance than a higher order one, regardless of what type of algorithm has been used to derive them.

- P and Q: The dark blue and the light blue columns suggested that, in either $2s$ or $9s$ after the fault, the lower order dynamic equivalent models (Model 2, Model 3 and Model 5) behave better than the higher order models (Model 4 and Model 6) and the constant power model.

As we can see from the results, by selecting proper orders, state-space dynamic equivalent models may have overall better performances than the constant power model in power, voltage and frequency at the connecting bus.

In addition, for dynamic equivalent models derived using both N4SID and PEM algorithms, the $4th$ order models behave better than the higher order ones. Such results may be very confusing since a higher order model should usually keep more dynamic features of a system in system reduction. A possible reason for this, as discussed in section 5.3.5, might be that the higher order models may involve more effects from the transmission network side than the lower order ones.

Under Faults at Different Locations



(a) Over 9s after the fault (b) Over 2s after the fault

Figure 8.3: Performance of dynamic equivalent models derived using Fault 1 in rotor angles of different machines (G1-G4) under different faults (Fault 1-Fault 5): Model 1, Constant power model; Model 2, 4^{th} order N4SID dynamic equivalent model; Model 3, 5^{th} order N4SID dynamic equivalent model; Model 4, 6^{th} order N4SID dynamic equivalent model; Model 5, 4^{th} order PEM dynamic equivalent model; Model 6, 5^{th} order PEM dynamic equivalent model.

Figure 8.3 shows the performance of different type of equivalent models (listed in Table 8.3) under faults at different locations (listed in Table 7.1). These equivalents were derived using Fault 1. The MSE in rotor angles of individual generators in the transmission network for each equivalent model was calculated and displayed in the figure.

For example, the first column from the left is the performance of Model 1, which is the constant power equivalent model. The dark blue layer (denoted as Fault 1 G1 in legend) of this column shows the MSE of the Model 1 in rotor angle response of machine G1 under Fault 1. Hence the column gives an overview of the performance of an equivalent model under different faults. Additional results can be found in Appendix F.

The observation from the results compares the general performances of these equivalent models:

- Rotor angle: Most of the dynamic equivalent models (except the $5th$ order PEM state-space model) are overwhelmingly better than the constant power model in rotor angle response of machines, especially in the first $2s$ after the fault was cleared.
- The lower order state-space models have better performance than the higher ones, regardless what algorithms were used to derive the state-space models.
- To derive a same order state-space model, N4SID algorithm is as good as, if not better than, PEM algorithm. This is interesting as the subspace identification algorithm (e.g.: N4SID) is regarded as an efficient but less accurate method than the PEM algorithm (see section 5.3.4).

We can also find out that, under a given fault, the equivalent performances in rotor angles of different machines are similar. This means that it is enough to look at just one machine to evaluate the response.

8.4 Disturbance used to Derive Dynamic Equivalent

8.4.1 Introduction

The dynamic equivalents are derived from time series collected after a disturbance. The performance of the dynamic equivalents may be affected by both the type and the location of the disturbances used to derive them. For some classical dynamic equivalencing methods like the coherency-based method, the accuracy of the equivalent is dependent on the disturbance. An equivalent derived from one disturbance may not perform well for another disturbance, which is regarded as the generalisation ability problem in dynamic equivalencing.

The dynamic equivalents derived in this project are based on the simulated time series at the connecting bus after a line fault located close to the connecting bus. The reason why a line fault close to the connecting bus was selected for system identification will be discussed in this section.

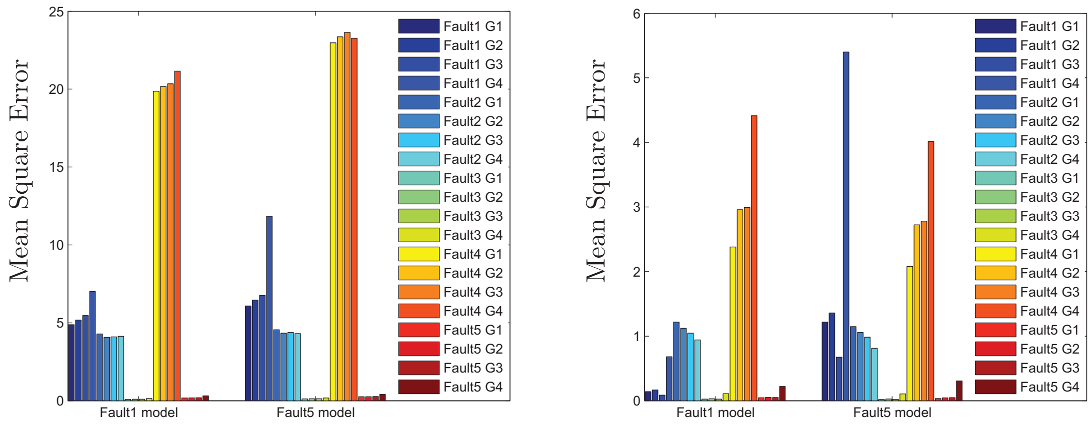
Equivalents derived from different disturbances have been studied for comparison. These disturbances include line faults located at different places in the power system, and some other types of disturbances besides line faults. Once an dynamic equivalent was derived, the responses of this equivalent model were simulated with the faults at different locations.

8.4.2 Simulation Results

All the dynamic equivalents mentioned before were derived from the responses of the original power system after a line fault (Fault 1). These dynamic equivalents were proved to perform well, and have better performances than the constant power model, under faults that occur at other locations.

The following discussion will compare the equivalent models derived with the system responses to various faults. The dynamic equivalent model structure used was the 4th-order N4SID state-space model.

Faults at Different Locations



(a) Over 9s after the fault

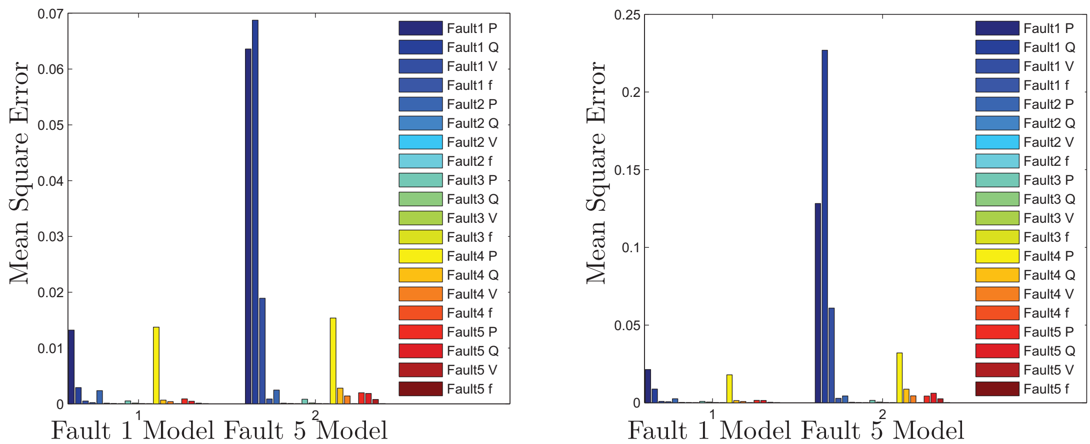
(b) Over 2s after the fault

Figure 8.4: Performance of $4th$ order N4SID dynamic equivalent model derived by different faults (Fault 1 and Fault 5) in rotor angles of different machines (G1-G4) under different faults (Fault 1-Fault 5): Fault 1 model, a dynamic equivalent derived using Fault 1; Fault 5 model, a dynamic equivalent derived using Fault 5.

Fault 5 (marked on Figure 6.1) is located further away than Fault 1 from the connecting bus. Two $4th$ order N4SID dynamic equivalent models were derived using Fault 1 and Fault 5 respectively. Figure 8.4 shows the performance of the two equivalent models under faults occurred at different locations (listed in Table 7.1). The MSE in rotor angles of individual generators in the transmission network for each equivalent model was calculated and displayed in the figure. The left column group (denoted as Fault 1 model) is the performance of the dynamic equivalent model derived using Fault 1 and the right column group (denoted as Fault 5 model) is the performance of the model derived using Fault 5.

This figure shows the performance of the two dynamic equivalents in rotor angle of each machine (from G1 to G4) during dynamic simulations. For example, the first column (in dark blue, denoted as Fault 1 G1 in legend) of Fault 5 model column group shows the MSE of the dynamic equivalent model derived using Fault 5, in rotor angle response of machine G1 under Fault 1.

We can see from the figures that, over either $2s$ or $9s$ response, for different machines under different faults, the rotor angle MSE of the Fault 1 model is usually smaller than that of the Fault 5 model which indicates the dynamic equivalent model derived using Fault 1 has generally better performance than that derived by Fault 5. Even for the responses under Fault 5, which has been used to derive the Fault 5 model, the Fault 1 model still has performance as good as, if not better than, the Fault 5 model. This suggests that it is reasonable to use a linear dynamic equivalent model derived by the fault close to the connecting bus to represent the distribution network, and use it for simulations under the faults that occur in other locations.



(a) Over 9s after the fault

(b) Over 2s after the fault

Figure 8.5: Performance of $4th$ order N4SID dynamic equivalent model derived by different faults (Fault 1 and Fault 5) in real power P, reactive power Q, voltage V and frequency f at the connecting bus under different faults (Fault 1 - Fault 5): Fault 1 model, a dynamic equivalent derived using Fault 1; Fault 5 model, a dynamic equivalent derived using Fault 5.

Figure 8.5 shows the MSE of the simulations at the connecting bus. We can see that, over either $2s$ or $9s$ responses, the Fault 1 model again has generally better performance than the Fault 5 model, even for the responses under Fault 5.

This can be regarded as an advantage of this dynamic equivalent since some dynamic equivalents described in the literature can only provide good performances under the disturbance which was used to derive it, but cannot do well under other disturbances.

Other Types of Disturbances

Besides what was derived by line fault in other locations, the dynamic equivalents derived by some other types of usual disturbances have also been implemented and tested.

- Load Rejection

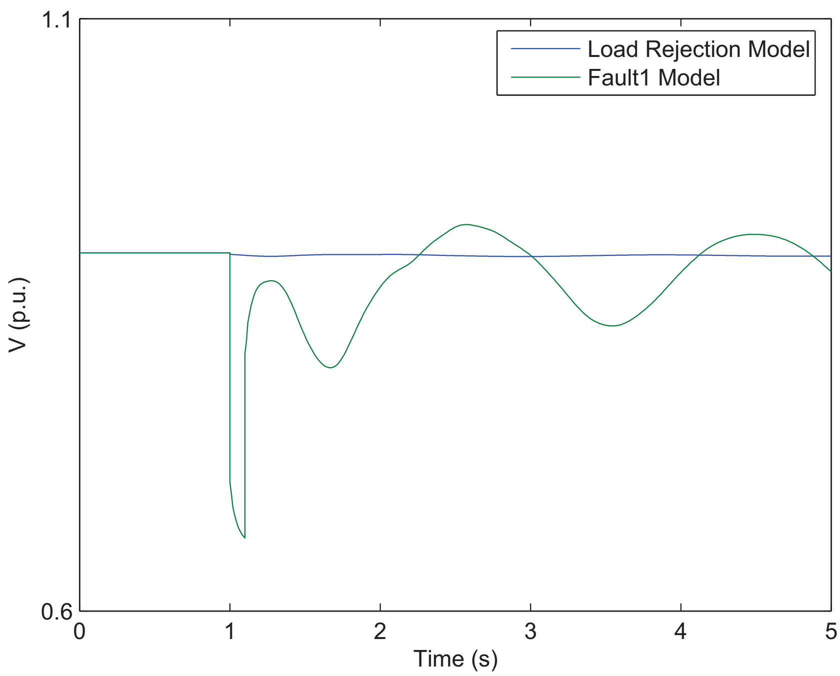


Figure 8.6: Dynamic response voltage V at the connecting bus under load rejection

A $10MVar$ reactive load was rejected from the connecting bus between the transmission and the distribution networks as a disturbance. This disturbance locates very close to Fault 1 but the load rejection normally causes smaller voltage variation at the connecting bus than the short-circuit as its response is usually the variation in power instead of in voltage [80]. Figure 8.6 shows the system response to the two disturbances in voltage at the connecting bus, in which the load rejection disturbance hardly caused oscillations.

Due to its lack of richness in the signal, load rejection is not suitable to be used as the disturbance to derive dynamic equivalent models. For testing purposes, a dynamic equivalent model of the distribution network has been derived using load rejection disturbance (denoted as load rejection model) and implemented into PSS/E to compare with the dynamic equivalent model derived using Fault 1 (denoted as Fault 1 model). The load rejection model led to instability of the power system under some simulated faults, which did not cause any instability in the original power system, while the Fault 1 model provides similar performance to the original distribution system.

This concludes that load rejection is not suitable to be used as input data to derive the dynamic equivalents.

- Line tripping

A simulated disturbance by tripping and re-closing the line, at which Fault 1 appeared, has been introduced to the power system model. Under this line tripping disturbance, the voltage variation at the connecting bus is still small (see Figure 8.7).

The dynamic equivalent model derived by this disturbance has been implemented and tested. It does not give as good result as those derived by line fault in the simulation. The performance of the dynamic equivalent model derived by tripping and reclosing a line (denoted as line tripping model in legend) and that derived by a Fault 1 (denoted as Fault 1 model in legend) are compared in Figure 8.8 and

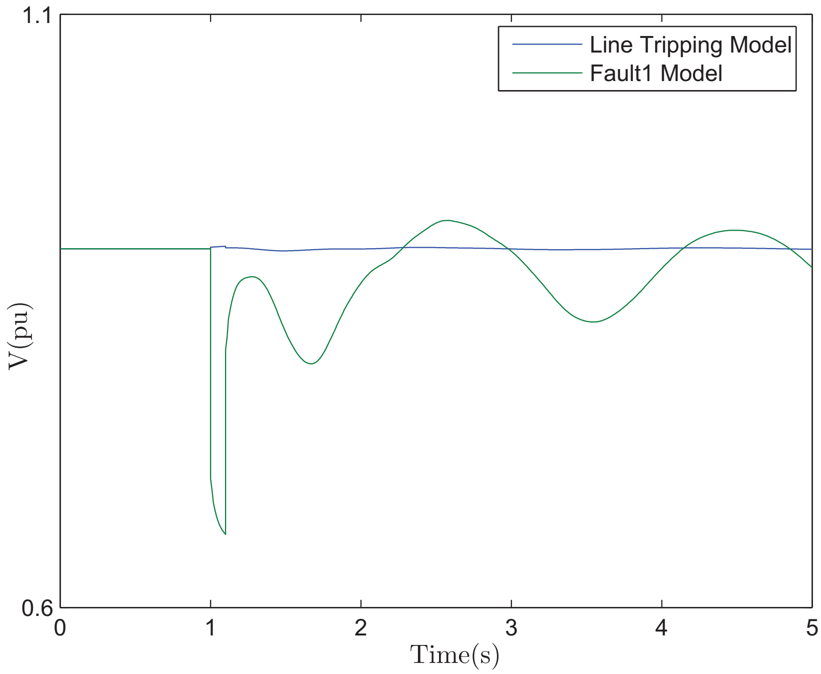


Figure 8.7: Dynamic response voltage V at the connecting bus after tripping and closing the line

Figure 8.9.

In Figure 8.8, the two dynamic equivalent models were tested under Fault 1. The figure shows their performances for rotor angle response of machine $G4$. An obvious advantage of Fault 1 Model can be seen in fitting the original system response under this disturbance.

In Figure 8.9, the two dynamic equivalent models were tested under the disturbance of tripping and reclosing a line, which was used to derive the line tripping model. The figure shows their performance for rotor angle response of machine $G3$.

A big difference can be seen in the performance of the line tripping model and that of the original distribution network. Fault 1 model however can fit the response of the original system very well.

Hence a conclusion can be drawn that the dynamic equivalent model derived using the simulated fault can have better performance than those derived using other types of disturbances. The advantage of the fault-derived

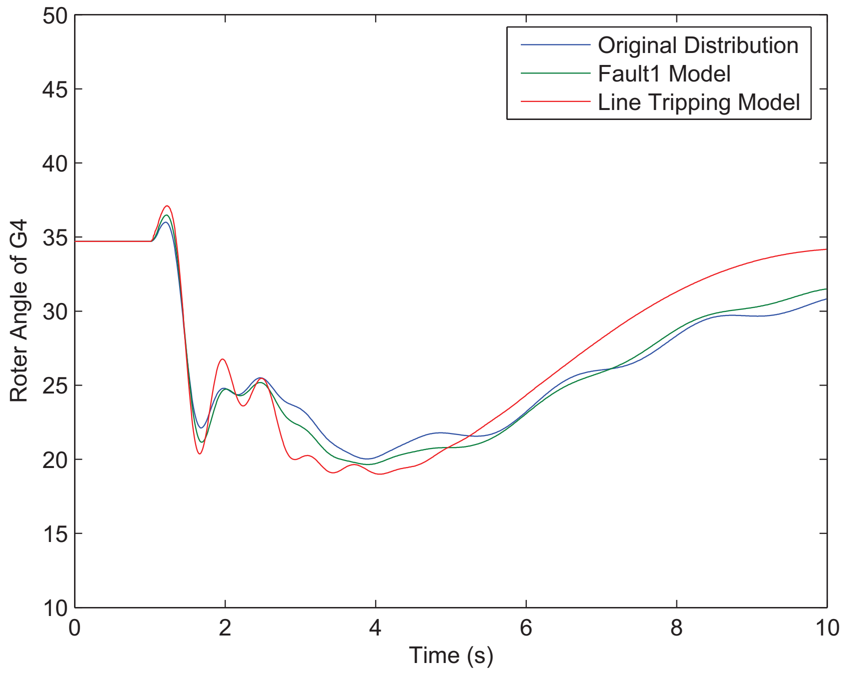


Figure 8.8: Dynamic responses of the dynamic equivalent models after Fault 1

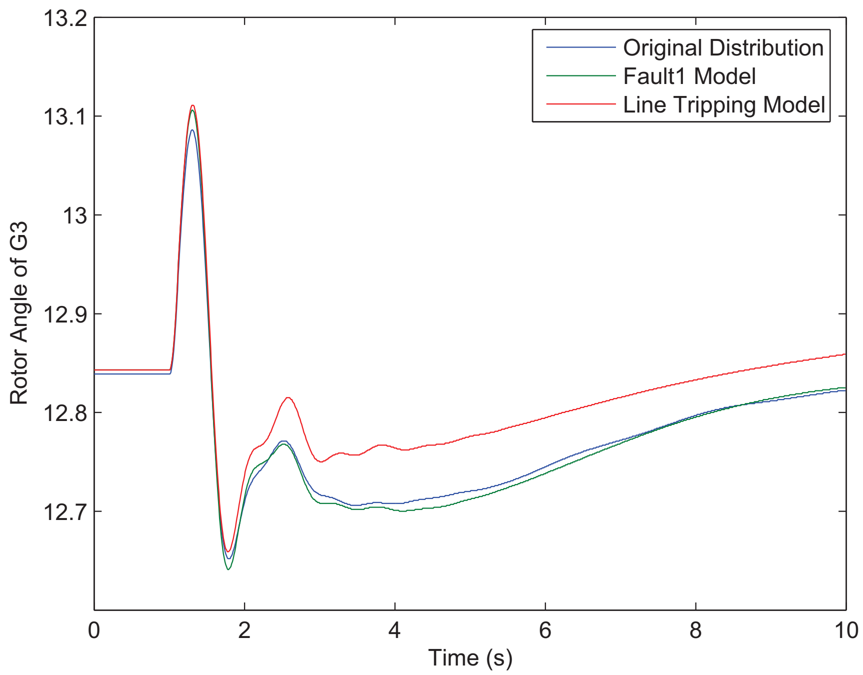


Figure 8.9: Dynamic responses of dynamic equivalent models after tripping and closing the line

dynamic equivalent can not only be seen under faults but also be seen under other types of disturbances.

8.4.3 Conclusion

Simulation results have shown that the dynamic equivalent models derived using a short circuit line fault close to the connecting bus can work better than those derived using other disturbances, either short-circuit faults located further from the connecting bus or other types of disturbances.

One possible reason for this is that in system identification the richness of the signal used to derive the model may have significant influence on the accuracy of the model. Hence a disturbance which can cause relatively large oscillations in the amplitude (e.g.: line fault) is suitable for deriving dynamic equivalents.

Also, for the reason mentioned in section 6.2.4, as we cannot isolate the distribution network from the transmission network during dynamic equivalencing, the dynamic equivalent we obtained may involve some of the dynamic response of transmission network. The disturbance close to the distribution network should be selected for dynamic equivalencing to have less transmission network response involved.

Hence a line fault close to the distribution network was proved to be an appropriate disturbance for deriving the dynamic equivalent of distribution network.

8.5 Distribution Network Size Influence

In this section, three distribution network models have been compared to analyse the distribution network size influence on the power system stability and equivalent model performance. Table 8.4 shows the generators embedded in a small distribution network (Figure 8.10). This small distribution network was then duplicated (including all the generators, branches and loads) and merged as a double-sized distribution network (denoted as 2 × *small* and shown in Figure 8.11). The triple-sized distribution network denoted as 3 × *small* was also created (see Figure 8.12).

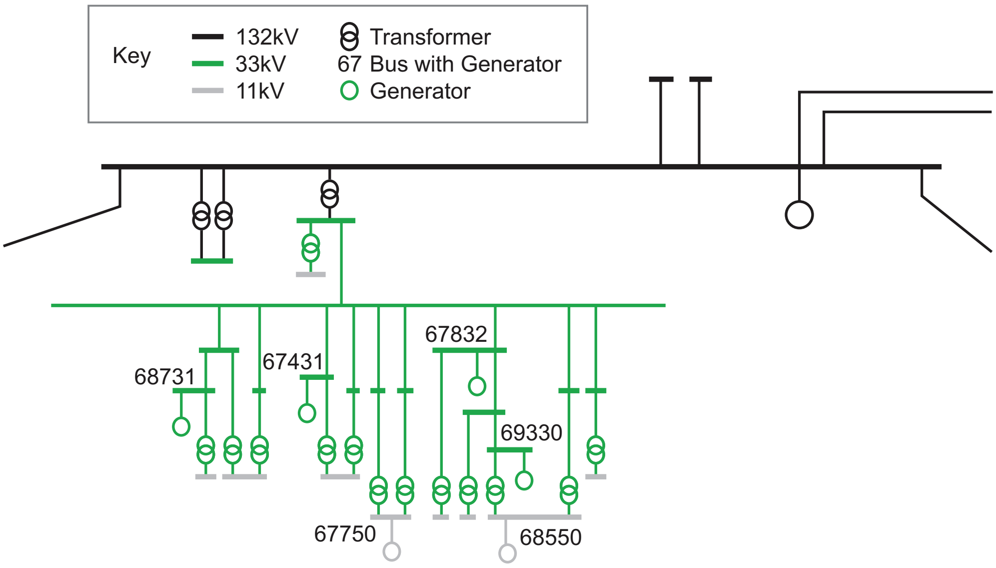


Figure 8.10: The *small* distribution network

Bus Number	Generator Type	Capacity (MW)
68731	Steam	25
69330	Steam	25
67431	Diesel	2
67832	Diesel	4
67750	Hydro	18
68550	Hydro	12

Table 8.4: Embedded small synchronous generators in the *small* distribution network

The disturbance used for testing is still Fault 1 located very close to the

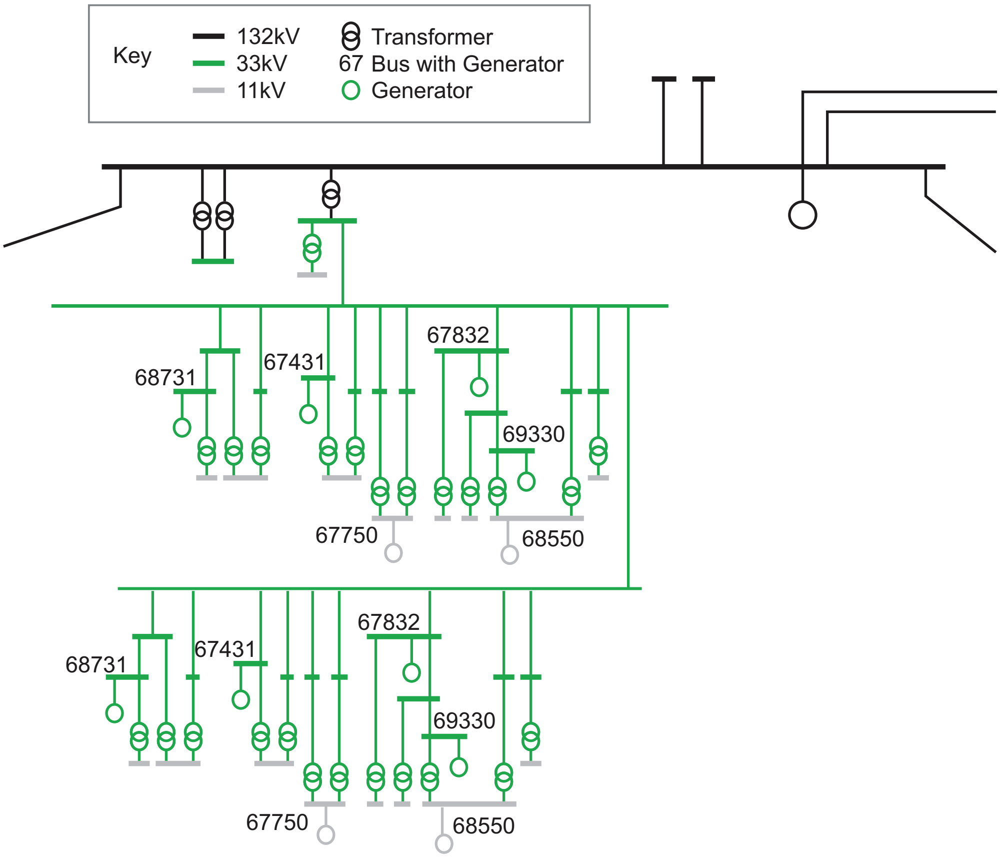


Figure 8.11: The 2 × *small* distribution network

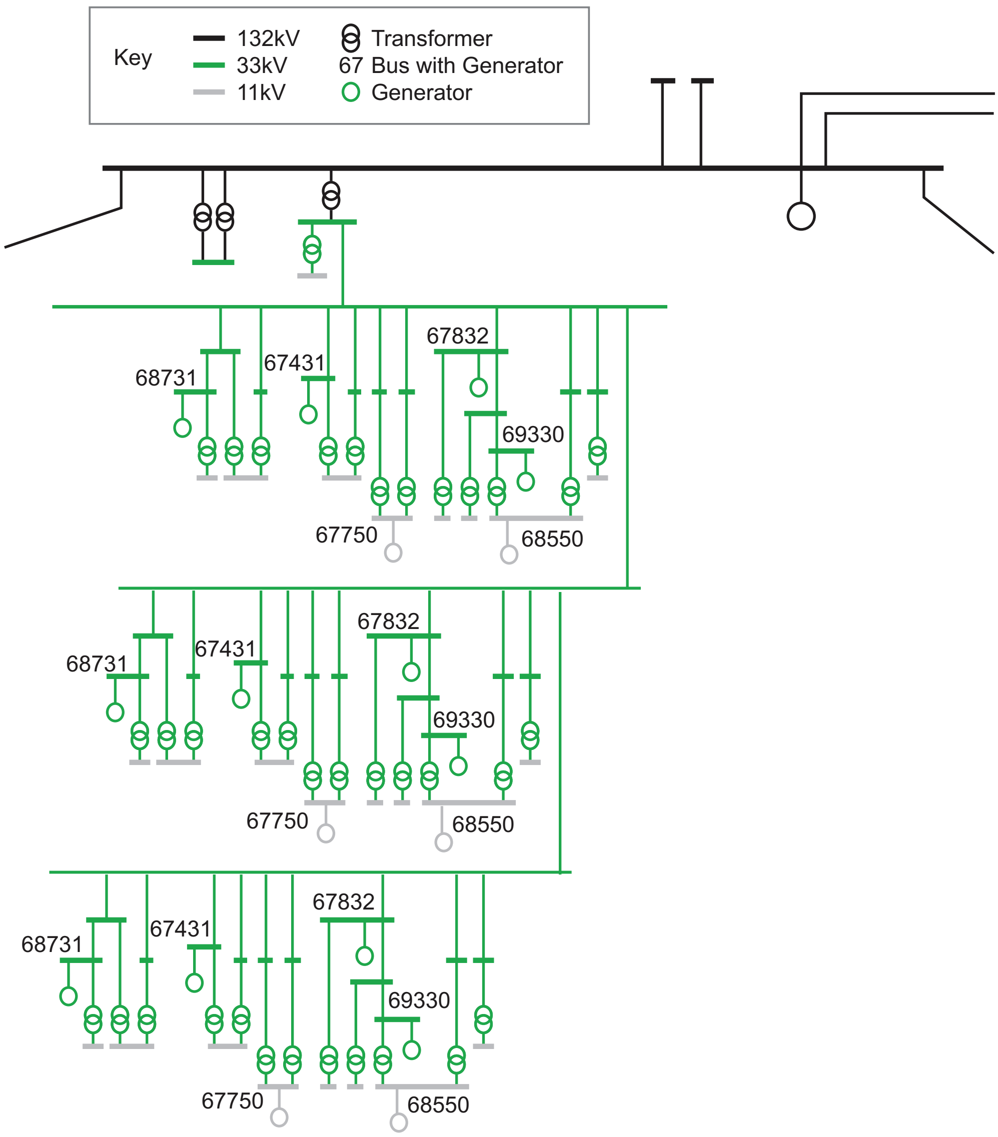


Figure 8.12: The 3 × *small* distribution network

connecting bus at the transmission system. In simulations, the fault was cleared in a time period (by tripping the line) shorter than the power system CCT to ensure that generators in the distribution network are stable.

For evaluation, the swing curves of the rotor angles for generators in the transmission network (with locations shown on Figure 6.1) are analysed. The rotor angles of four different generators (from G1 to G4, see Figure 6.1) in the transmission network are selected as outputs.

Distribution Network	Model	G1 MSE	G2 MSE	G3 MSE	G4 MSE
small	Model 1	0.038	0.054	0.196	0.650
small	Model 2	0.059	0.093	0.225	1.371
small	Model 4	0.027	0.084	0.261	3.129
$2 \times$ *small*	Model 1	0.311	0.325	0.267	1.613
$2 \times$ *small*	Model 2	0.142	0.170	0.287	1.096
$2 \times$ *small*	Model 4	0.201	0.373	0.351	2.065
$3 \times$ *small*	Model 1	4.801	5.260	4.831	5.565
$3 \times$ *small*	Model 2	0.260	0.455	0.718	3.626
$3 \times$ *small*	Model 4	1.365	1.913	2.716	10.46

Table 8.5: Rotor angle MSE of equivalent models over $2s$ after Fault 1 was cleared: Model 1, Constant power model; Model 2, 4^{th} order N4SID dynamic equivalent model; Model 4, 6^{th} order N4SID dynamic equivalent model.

Table 8.5 shows the performance of the equivalent models for three different sized distribution networks in rotor angle MSE under Fault 1 (see Table 7.1 and Figure 6.1). The dynamic equivalent models are the single-input state-space models listed in Table 8.3. The rotor angle MSEs were calculated using the time domain response over $2s$ after the fault.

- From the responses of the power system with the D&G distribution network we can see that:

  - For the D&G distribution network, the constant power equivalent model (Model 1) has generally better performance than all the dynamic equivalent models.
  - The $6th$ order dynamic equivalent (Model 4) is better for generators farther from the fault than the $4th$ order dynamic equivalent (Model 2). The latter however is better for generators close to the fault.

- From the responses of the power system in the double-sized D&G distribution network we can see that:
  - With the size of distribution system increasing, both the constant power model (Model 1) and the equivalent models become worse in their performance.
  - However, the advantage of dynamic equivalent models becomes slightly obvious. The $4th$ order state space dynamic equivalent model (Model 2) is better than constant power equivalent model (Model 1) in simulating the rotor angle responses of most generators.
  - The $4th$ order dynamic equivalent model (Model 2) performs better than the $6th$ order dynamic equivalent model (Model 4).
- From the responses of the power system with the triple-sized D&G distribution network we can see that:
  - With the size of distribution network increasing further, both the constant power equivalent model (Model 1) and the dynamic equivalent models become worse in performance than with a smaller distribution network.
  - In this case, the dynamic equivalent models perform better than the constant power equivalent model (Model 1) for all generators

in the transmission network.

– The results confirm that with the distribution network size increasing, the advantage of the dynamic equivalent model to the constant power equivalent model becomes more obvious.

– The $4th$ order dynamic equivalent model (Model 2) performs better than the $6th$ order dynamic equivalent model (Model 4).

8.6 Type of Embedded Generators

The discussions made so far are for the case that the distribution network is embedded with synchronous machines. With the rapid development of renewable technologies, renewable generators such as wind generators (e.g.: DFIG) have also been implemented into distribution networks.

The following discussion will be on the influence of the type of the generators embedded in the distribution network on the performance of the dynamic equivalent models. Prony analysis will be applied for this discussion.

8.6.1 Embedded Conventional Generation

In this subsection, the external system is the distribution network (Figure 8.12) embedded with small synchronous machines (listed in Table 6.2).

Machine G1 Response under Fault 1

As shown in the map (Figure 6.1), generator $G1$ is located far from either the distribution network or the disturbance Fault 1. The real power P generated by $G1$ is $230.33MW$. For different equivalent models, the dynamic response of rotor angle of $G1$ under Fault 1 was plotted in Figure 8.13. From the above dynamic responses, the identified Prony modes for power system

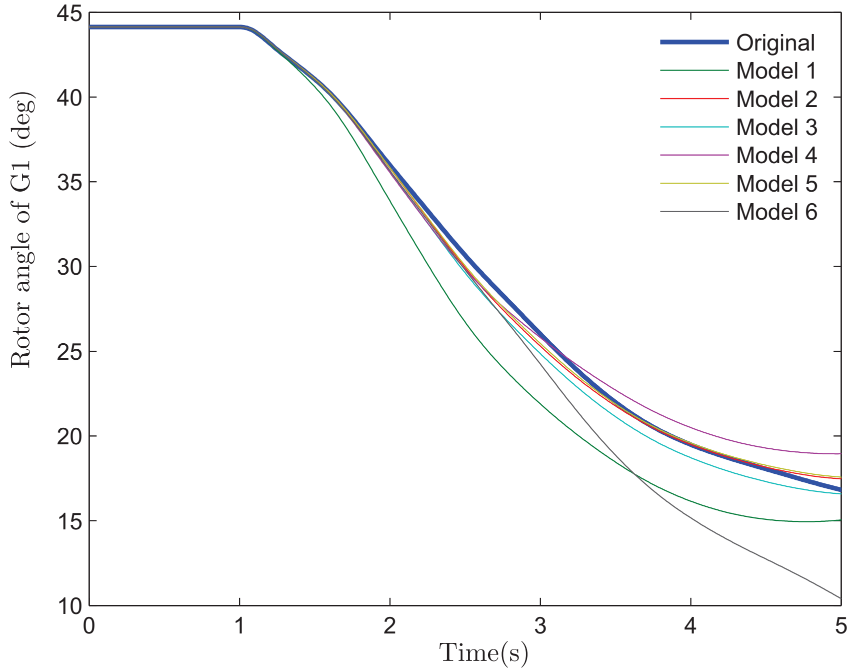


Figure 8.13: Dynamic response of rotor angle of $G1$ under Fault 1

based on different dynamic equivalents are shown in Figure 8.14. In section 7.6, we have introduced the categories of the modes in the power system. Hence we can group the modes in the figure. The column graphs in Figure 8.15 show the difference of the models in percentage from the original system. Some insights are listed as follows:

- Interarea modes

  In theory, the major difference between the constant power model and the original distribution network model should be in the interarea modes. According to the frequency level, the first circle from the real axis marked the interarea modes (mode 1) in the figure.

  For the interarea mode in Figure 8.14, as we can see, the damping difference for the constant power model is obviously higher than the low order dynamic equivalent models. A corresponding shift can be seen in the mode position.

  This is because Fault 1 is quite close to the connecting bus and

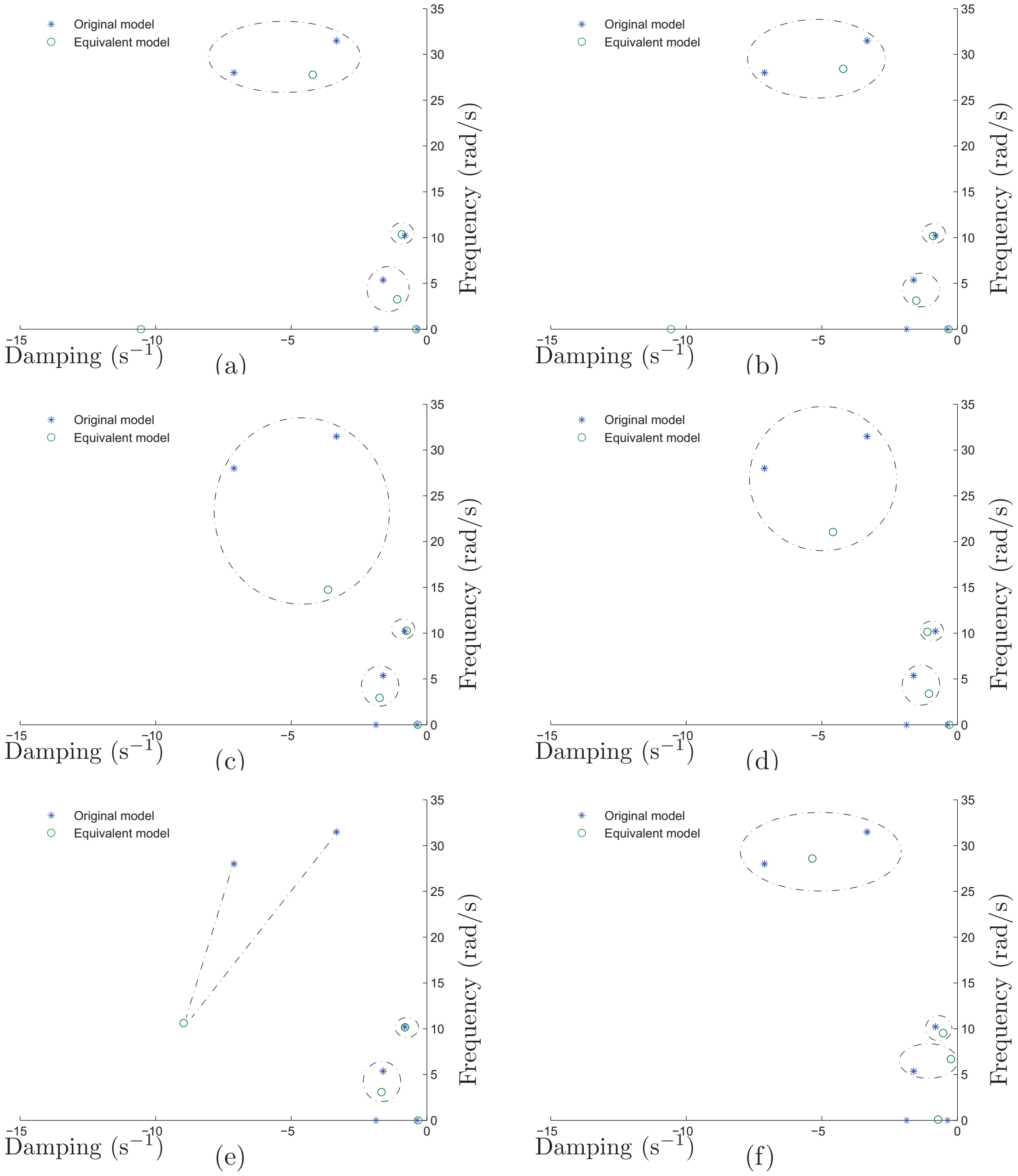


Figure 8.14: Rotor angle of Machine G1 under Fault 1 when using equivalent models derived using Fault 1 - Identified modes by Prony analysis: (a) Constant power model; (b) 4^{th} order N4SID dynamic equivalent model; (c) 5^{th} order N4SID dynamic equivalent model; (d) 6^{th} order N4SID dynamic equivalent model; (e) 4^{th} order PEM dynamic equivalent model; (f) 5^{th} order PEM dynamic equivalent model.

therefore causes large interarea oscillations. The constant power model, due to its lack of interarea modes of the distribution network, will lead to obvious mode shifts. Dynamic equivalent models may have smaller mode shift since they can represent the interarea modes of the distribution system.

As shown in Figure 8.15, all the equivalent models (Model 1-6) have similar performance in presenting the frequency of the interarea mode, however, with an obvious shift. The lower order dynamic equivalent models (Model 2, 3, 5) have better performance in presenting the damping of interarea mode than the constant power model (Model 1) and the higher order dynamic equivalent model (Model 4, 6). There are big differences in mode residuals due to the modes shift.

- Machine modes

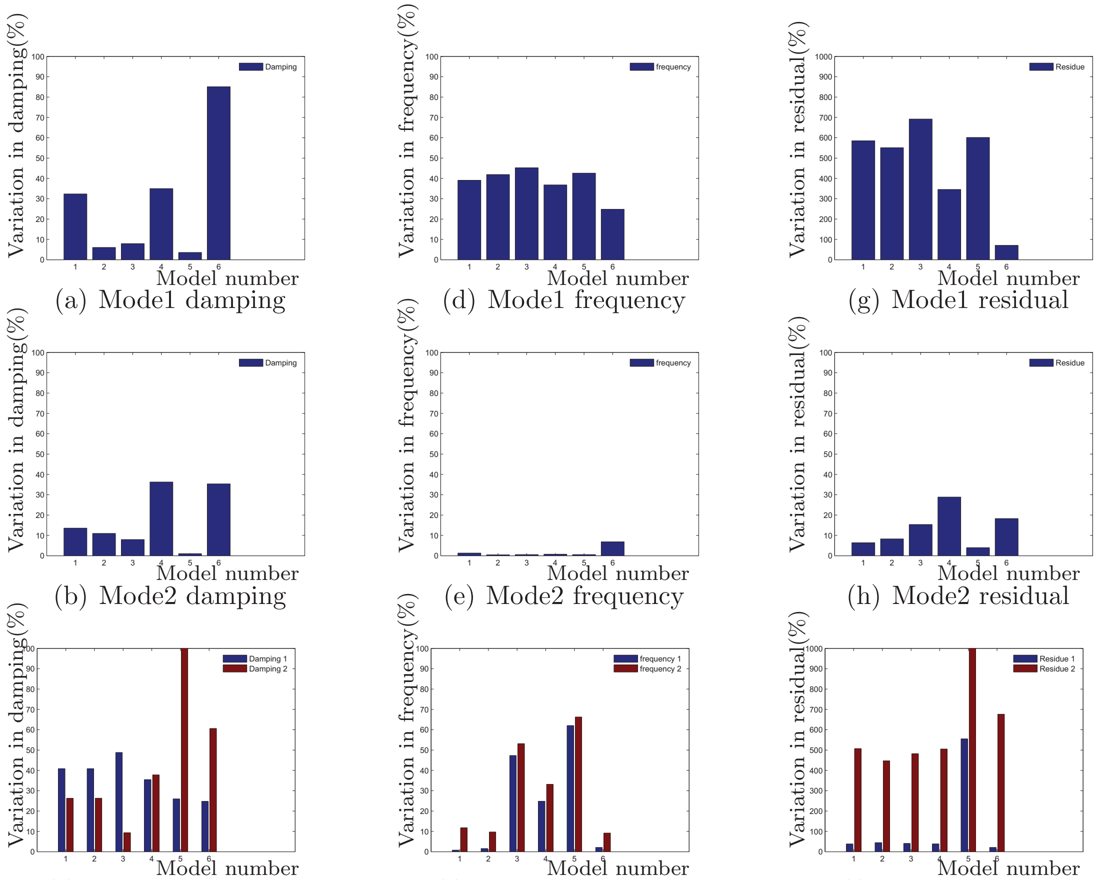
  The second circle marks the machine modes (mode 2). The frequency difference of the constant power and the lower order dynamic equivalent models are similar. The higher order dynamic equivalent models have a larger difference for the machine mode.

  Figure 8.15 shows that the constant power model (Model 1) and the lower order dynamic equivalent model (Model 2, 3, 5) have good performance in presenting the damping, frequency and residual of the machine modes, whilst the higher order dynamic equivalent models have poor performance in presenting the damping of the machine modes.

  This again is because the higher order state-space dynamic equivalent model may involve the machine modes from the transmission network when they are being derived.

- High frequency modes

  High frequency modes (mode 3) may shift largely on the s-plane. A small change in the system can cause a big difference in high frequency mode positions.



(a) Mode1 damping (d) Mode1 frequency (g) Mode1 residual

(b) Mode2 damping (e) Mode2 frequency (h) Mode2 residual

(c) Mode3 damping (f) Mode3 frequency (i) Mode3 residual

Figure 8.15: Rotor angle of Machine G1 under Fault 1 when using the equivalent models derived using Fault 1 - Variation in damping, frequency and residual of modes identified by Prony analysis: Model 1, Constant power model; Model 2, 4^{th} order N4SID dynamic equivalent model; Model 3, 5^{th} order N4SID dynamic equivalent model; Model 4, 6^{th} order N4SID dynamic equivalent model; Model 5, 4^{th} order PEM dynamic equivalent model; Model 6, 5^{th} order PEM dynamic equivalent model.

Machine G1 Response under Fault 2

The modes identified from the rotor angle response of $G1$ under Fault 2 (see Figure 8.16) are shown in Figure 8.17.

For the interarea mode in Figure 8.17, unlike what happened in Figure 8.14, no significant damping or frequency difference for the constant power model can be seen. This is clearly visible in Figure 8.18. The interarea modes locate relatively close to those of the original system.

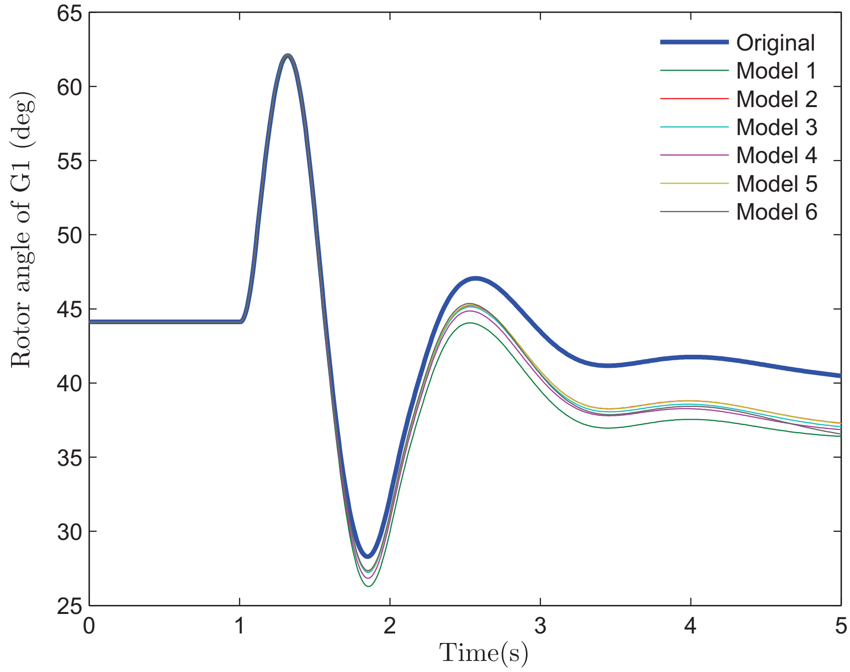


Figure 8.16: Dynamic response of rotor angle of $G1$ under Fault 2

This is because Fault 2 is located much further away than Fault 1 from the distribution network. Hence the interarea mode oscillations between the transmission network and the distribution network caused by Fault 2 are very small. In other words, the impact of the distribution network on the transmission network dynamic response caused by Fault 2 was small.

Hence for Fault 2, the constant power model has better performance in representing the power system interarea mode than the dynamic equivalent models, as the dynamic equivalent models may duplicate some response from oscillations in transmission network when they were derived.

Machine G2 Response under Fault 1

As shown in the map (Figure 6.1), generator $G2$ is far from either the distribution network or the disturbance Fault 1. $G2$ has real power output P at $566.5MW$.

The modes identified by Prony analysis from the rotor angle response of $G2$

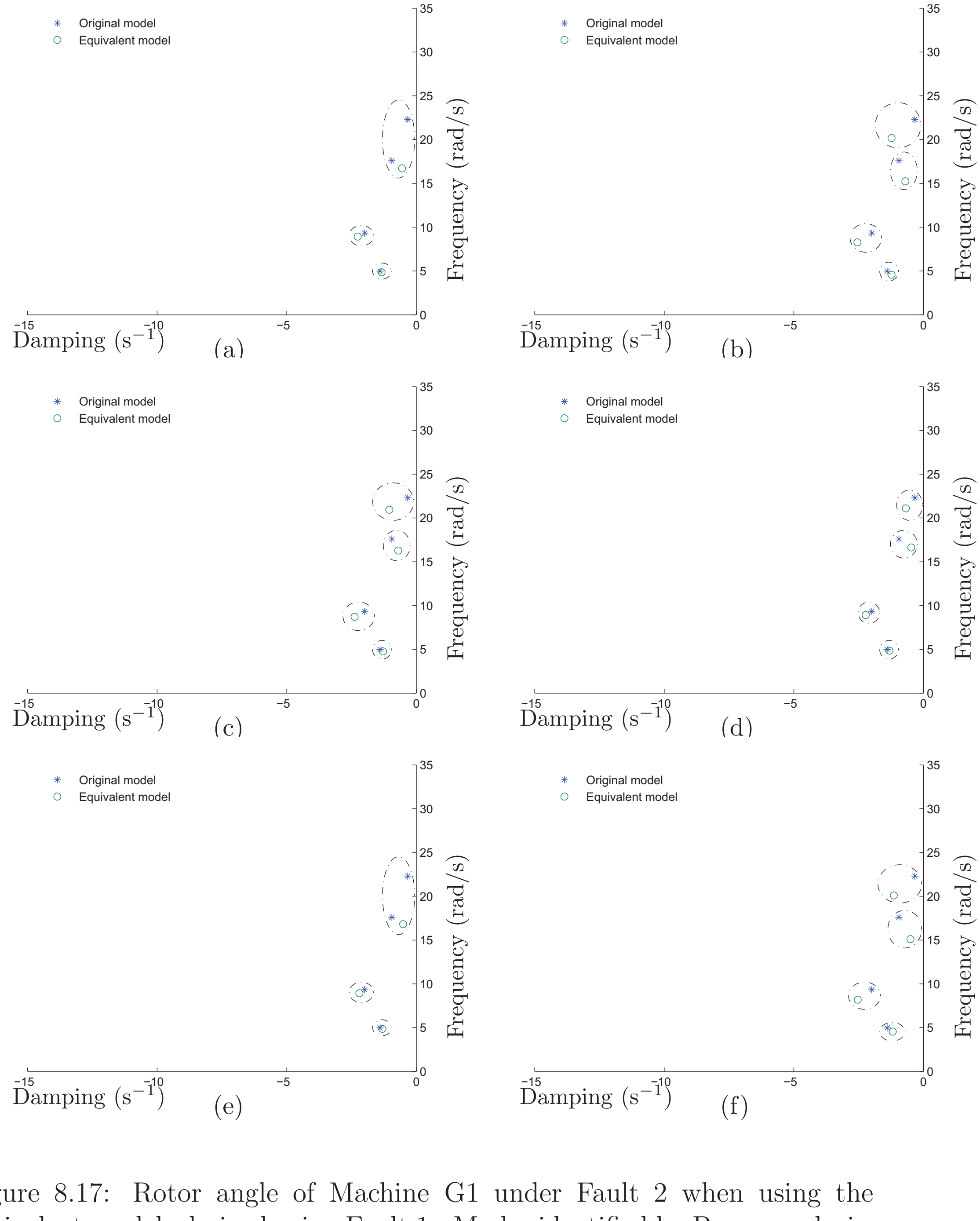
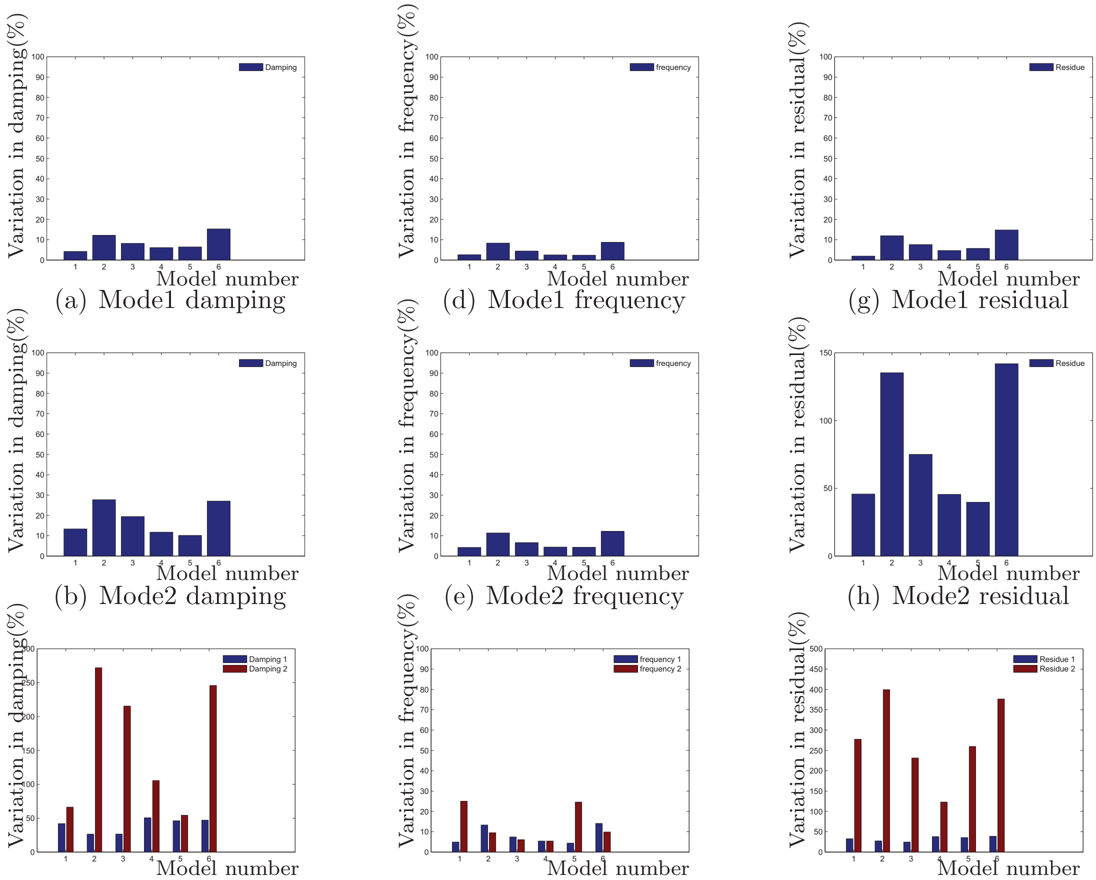


Figure 8.17: Rotor angle of Machine G1 under Fault 2 when using the equivalent models derived using Fault 1 - Modes identified by Prony analysis: (a) Constant power model; (b) 4^{th} order N4SID dynamic equivalent model; (c) 5^{th} order N4SID dynamic equivalent model; (d) 6^{th} order N4SID dynamic equivalent model; (e) 4^{th} order PEM dynamic equivalent model; (f) 5^{th} order PEM dynamic equivalent model.



(a) Mode1 damping (d) Mode1 frequency (g) Mode1 residual

(b) Mode2 damping (e) Mode2 frequency (h) Mode2 residual

(c) Mode3 damping (f) Mode3 frequency (i) Mode3 residual

Figure 8.18: Rotor angle of Machine G1 under Fault 2 when using the equivalent models derived using Fault 1 - Variation in damping, frequency and residual of modes identified by Prony analysis: Model 1, Constant power model; Model 2, 4^{th} order N4SID dynamic equivalent model; Model 3, 5^{th} order N4SID dynamic equivalent model; Model 4, 6^{th} order N4SID dynamic equivalent model; Model 5, 4^{th} order PEM dynamic equivalent model; Model 6, 5^{th} order PEM dynamic equivalent model.

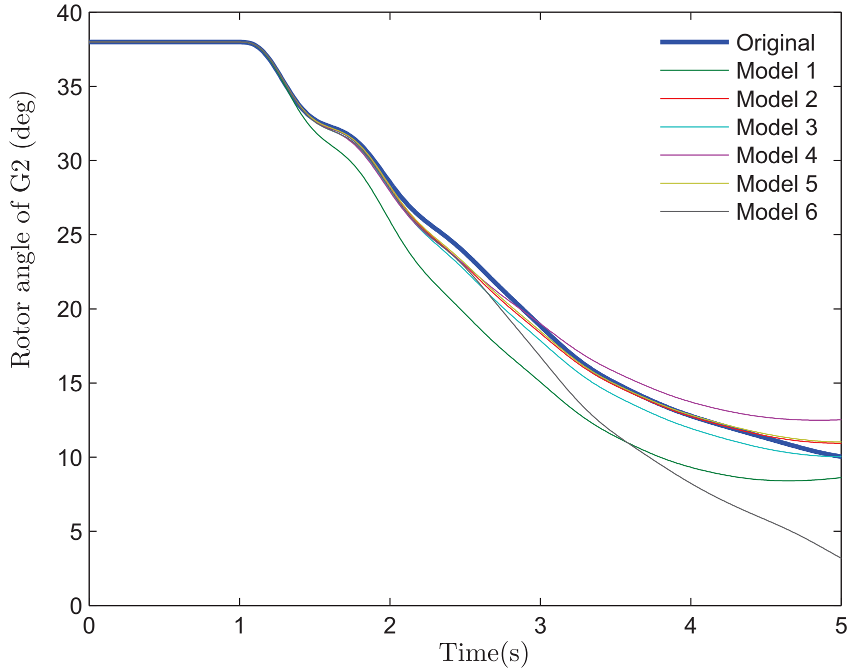


Figure 8.19: Dynamic response of rotor angle of $G2$ under Fault 1

under Fault 1 (see Figure 8.19) are shown in Figure 8.20 and Figure 8.21:

- Interarea and machine modes

  In Figure 8.20, machine modes and interarea modes of power system are represented as one mode by equivalent models. Hence the Mode 1 and Mode 2 circles become one. The Mode 1 and Mode 2 of constant power model are similar to those of the lower order dynamic equivalent models.

- High frequency modes

  The high frequency modes of the constant power model in Figure 8.20 shift further than those of the dynamic equivalent models.

Machine G3 Response under Fault 1

$G3$ is a small generator at a middle distance from the distribution network. The real power output of $G3$ is $11MW$. The modes identified by Prony

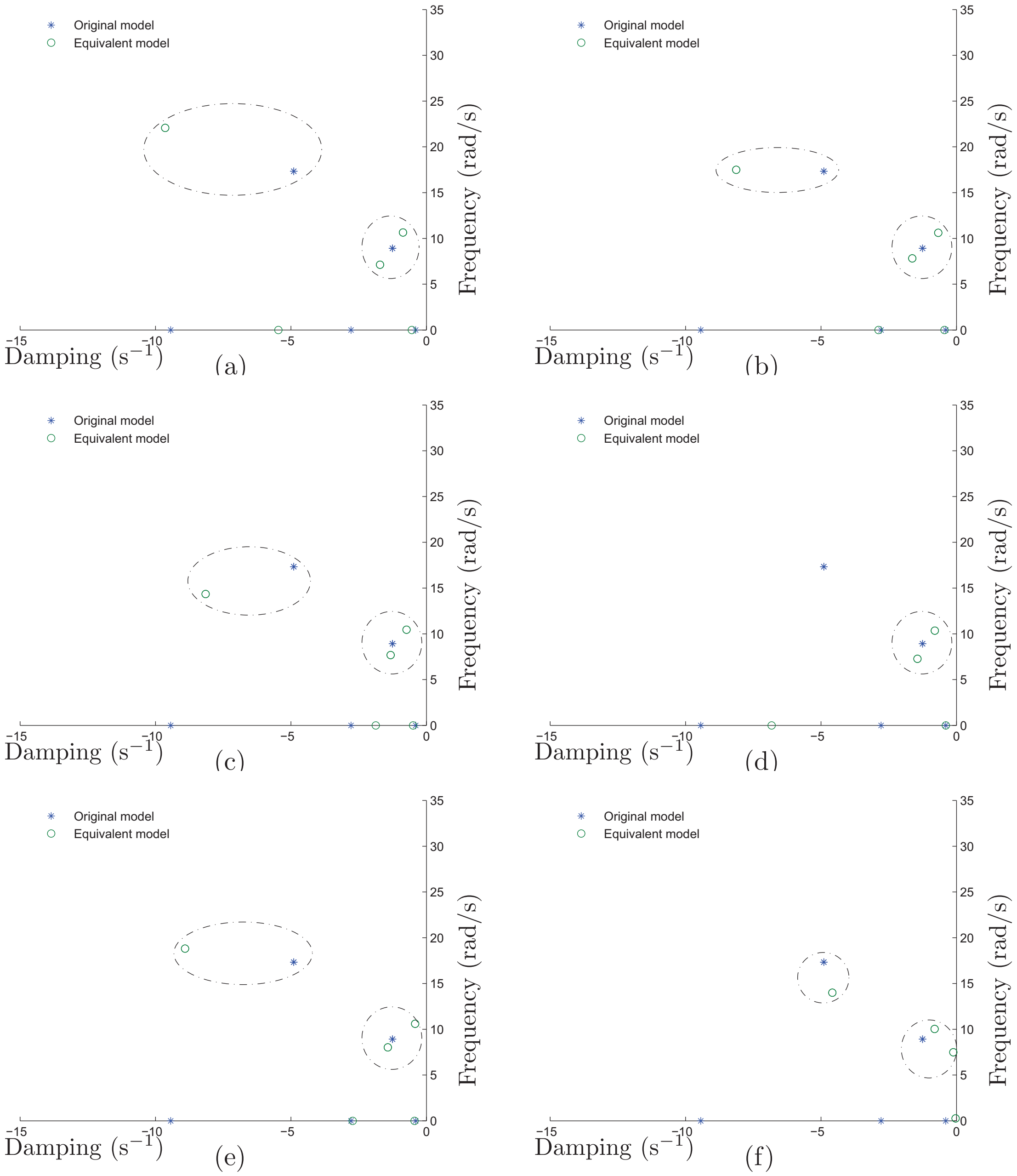
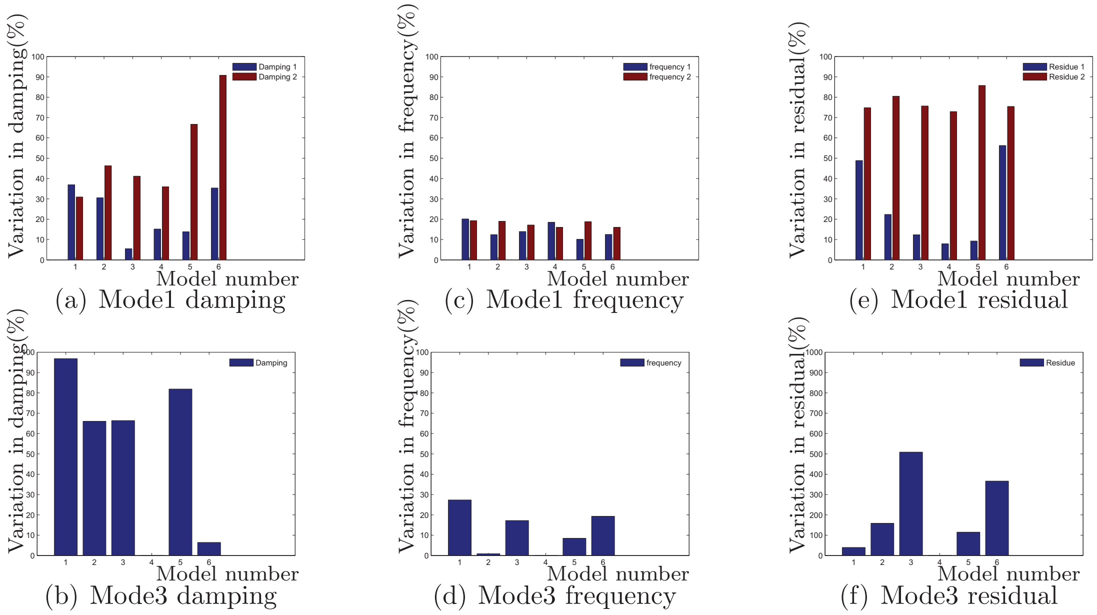


Figure 8.20: Rotor angle of Machine G2 under Fault 1 when using the equivalent models derived using Fault 1 - Modes identified by Prony analysis: (a) Constant power model; (b) 4^{th} order N4SID dynamic equivalent model; (c) 5^{th} order N4SID dynamic equivalent model; (d) 6^{th} order N4SID dynamic equivalent model; (e) 4^{th} order PEM dynamic equivalent model; (f) 5^{th} order PEM dynamic equivalent model.



(a) Mode1 damping (c) Mode1 frequency (e) Mode1 residual

(b) Mode3 damping (d) Mode3 frequency (f) Mode3 residual

Figure 8.21: Rotor angle of Machine G2 under Fault 1 when using the equivalent models derived using Fault 1 - Variation in damping, frequency and residual of modes identified by Prony analysis: Model 1, Constant power model; Model 2, 4^{th} order N4SID dynamic equivalent model; Model 3, 5^{th} order N4SID dynamic equivalent model; Model 4, 6^{th} order N4SID dynamic equivalent model; Model 5, 4^{th} order PEM dynamic equivalent model; Model 6, 5^{th} order PEM dynamic equivalent model.

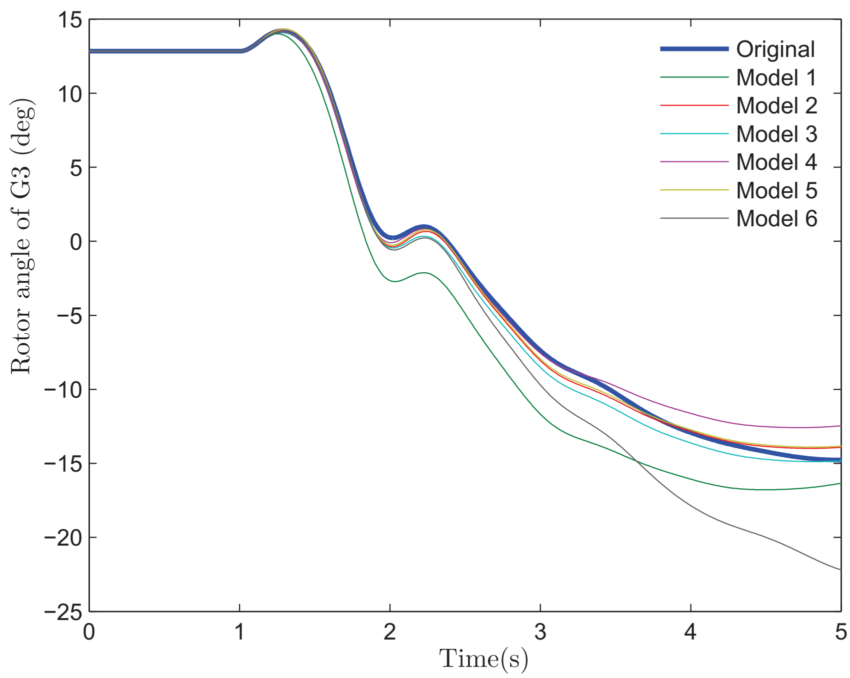


Figure 8.22: Dynamic response of rotor angle of $G3$ under Fault 1

analysis from the rotor angle response of $G3$ under Fault 1 (see Figure 8.22) are shown in Figure 8.23.

$G3$ is neither close to Fault 1 nor to the distribution system. Hence the mode positions of the constant power and most dynamic equivalent models are quite similar to the original system. However, a slight advantage of the low order dynamic equivalent can still be seen in Figure 8.24:

- Interarea modes
  Lower order dynamic equivalents can present the interarea modes better, especially in damping of the interarea modes.

- Machine modes
  All the equivalents can identify the position of machine modes well. Dynamic equivalent models, especially lower order ones present the residual of the machine mode in slightly better way.

- High frequency modes
  The constant power equivalent did not reproduce the high frequency modes at all.

Machine G4 Response under Fault 1

Generator $G4$ is quite close to the distribution network and the disturbance, and is a small generator with real power output $24.5MW$. The modes identified by Prony analysis from the rotor angle response of $G4$ under Fault 1 (see Figure 8.25) are shown in Figure 8.26. Figure 8.26 and Figure 8.27 show that:

- Interarea modes
  In Figure 8.26 graph (f), the interarea mode is not identified, but the machine mode is shifted because of the interarea oscillation. The equivalents are poor at representing the correct mode positions. This

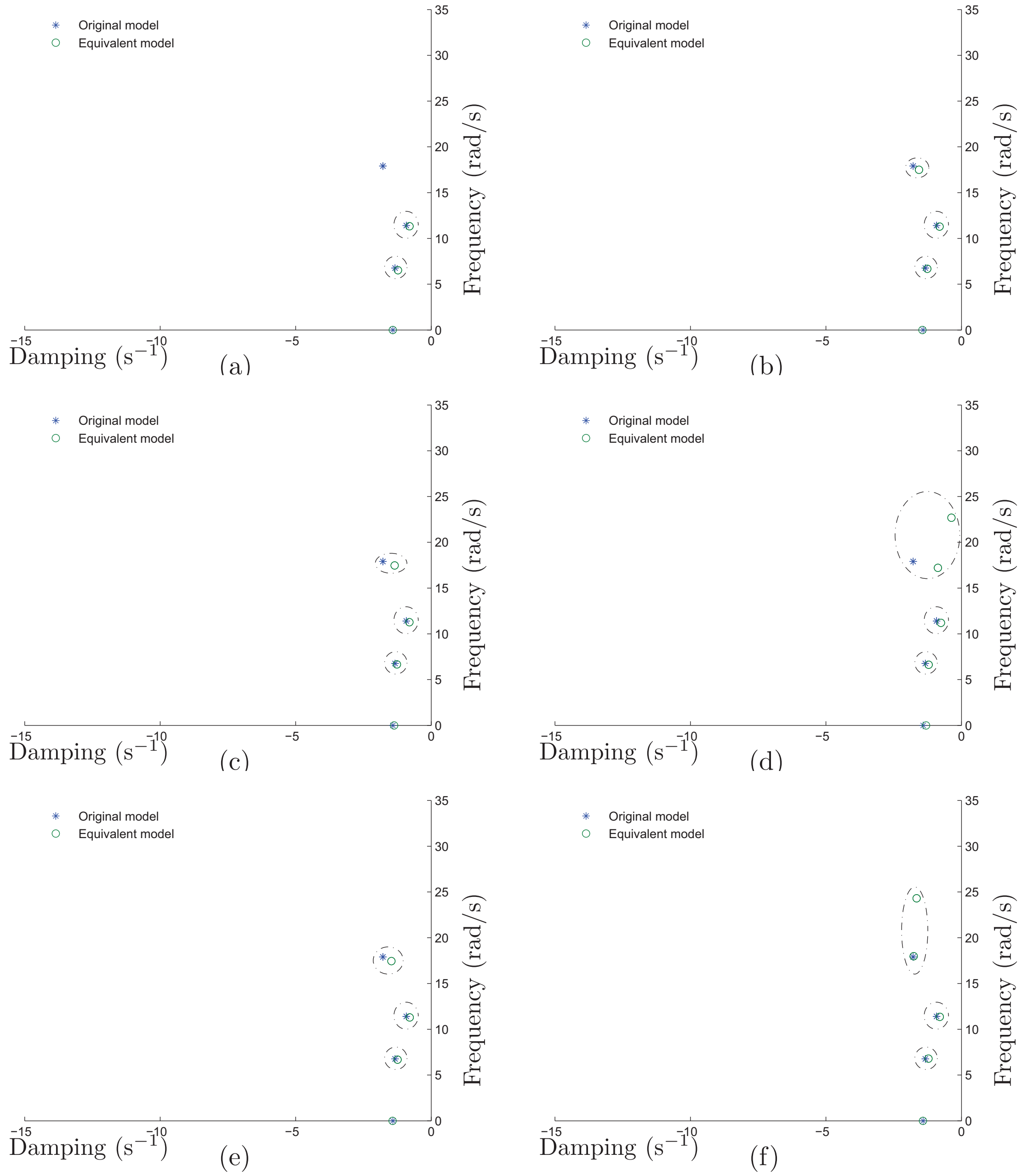
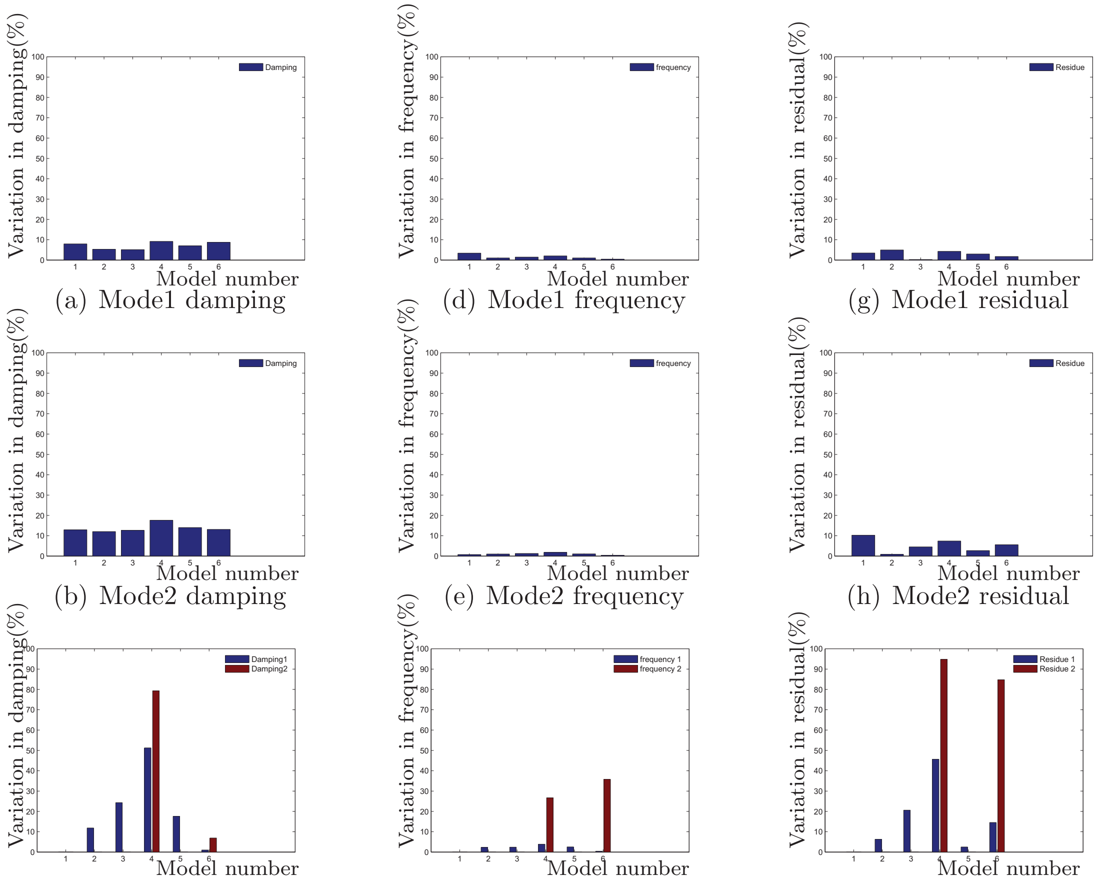


Figure 8.23: Rotor angle of Machine G3 under Fault 1 when using the equivalent models derived using Fault 1 - Modes identified by Prony analysis: (a) Constant power model; (b) 4^{th} order N4SID dynamic equivalent model; (c) 5^{th} order N4SID dynamic equivalent model; (d) 6^{th} order N4SID dynamic equivalent model; (e) 4^{th} order PEM dynamic equivalent model; (f) 5^{th} order PEM dynamic equivalent model.



(a) Mode1 damping (d) Mode1 frequency (g) Mode1 residual

(b) Mode2 damping (e) Mode2 frequency (h) Mode2 residual

(c) Mode3 damping (f) Mode3 frequency (i) Mode3 residual

Figure 8.24: Rotor angle of Machine G3 under Fault 1 when using the equivalent models derived using Fault 1 - Variation in damping, frequency and residual of modes identified by Prony analysis: Model 1, Constant power model; Model 2, 4^{th} order N4SID dynamic equivalent model; Model 3, 5^{th} order N4SID dynamic equivalent model; Model 4, 6^{th} order N4SID dynamic equivalent model; Model 5, 4^{th} order PEM dynamic equivalent model; Model 6, 5^{th} order PEM dynamic equivalent model.

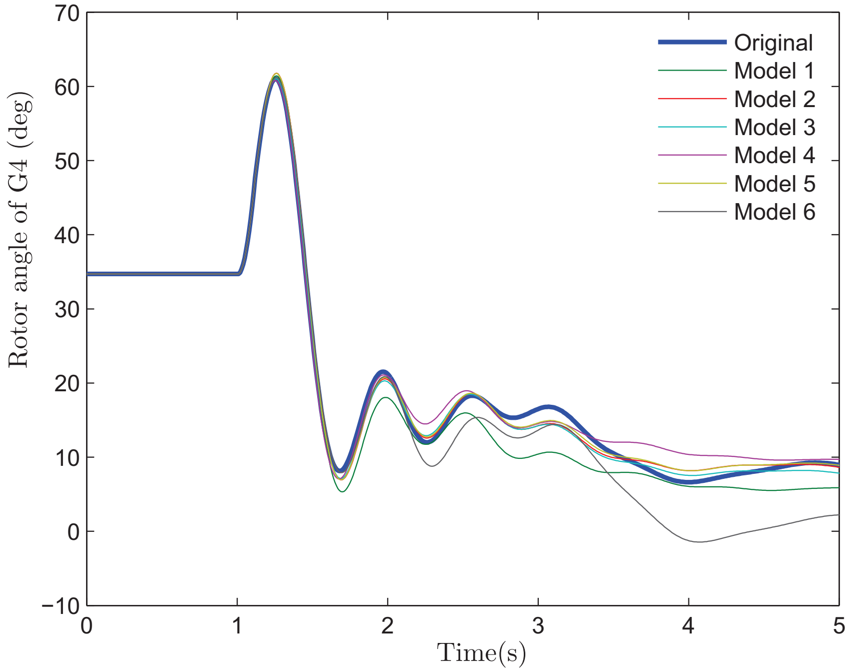


Figure 8.25: Dynamic response of rotor angle of $G4$ under Fault 1

might be because $G4$ is quite close to both the distribution system and the fault. It appears less reasonable to represent the distribution system as a linear model in this case.

- Machine modes

  Lower order dynamic equivalent models, especially the $4th$ order N4SID model (Model 2) have an advantage in representing the correct machine modes.

- High frequency modes

  The N4SID dynamic equivalents, especially the lower order ones, have overwhelming better performance in presenting the high frequency modes than the constant power and PEM dynamic equivalent models.

Conclusion

Additional results can be seen in Appendix G. Some conclusions can be drawn from this set of simulation results.

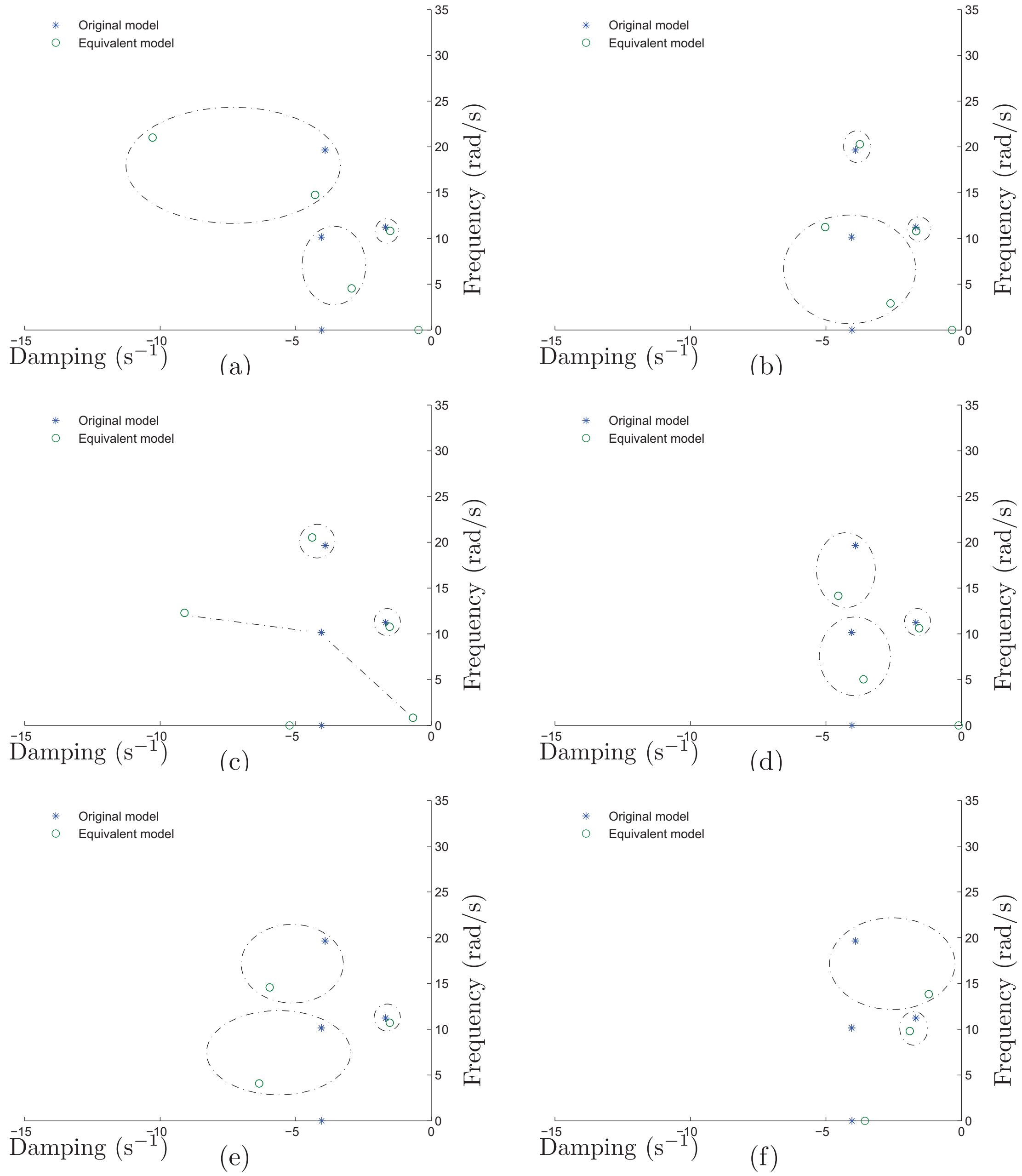
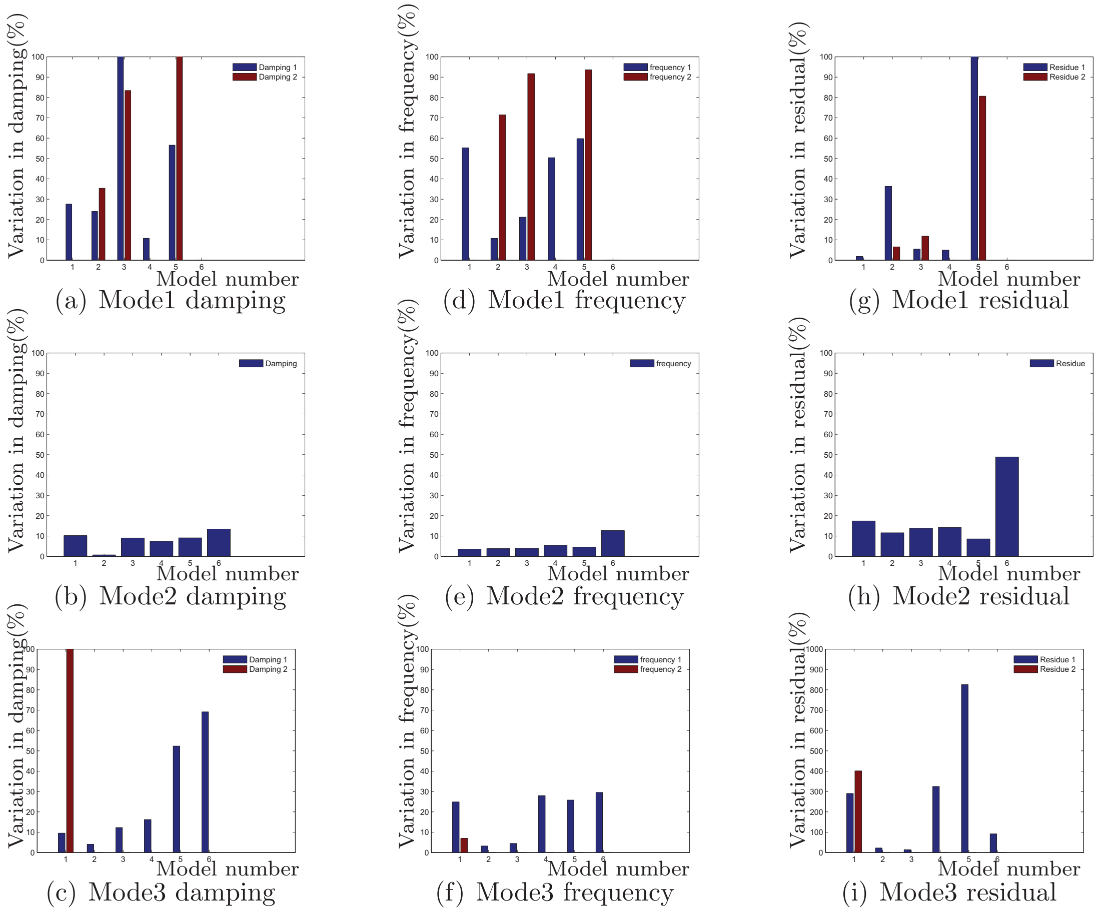


Figure 8.26: Rotor angle of Machine G4 under Fault 1 when using the equivalent models derived using Fault 1 - Modes identified by Prony analysis: (a) Constant power model; (b) 4^{th} order N4SID dynamic equivalent model; (c) 5^{th} order N4SID dynamic equivalent model; (d) 6^{th} order N4SID dynamic equivalent model; (e) 4^{th} order PEM dynamic equivalent model; (f) 5^{th} order PEM dynamic equivalent model.



(a) Mode1 damping (d) Mode1 frequency (g) Mode1 residual

(b) Mode2 damping (e) Mode2 frequency (h) Mode2 residual

(c) Mode3 damping (f) Mode3 frequency (i) Mode3 residual

Figure 8.27: Rotor angle of Machine G4 under Fault 1 when using the equivalent models derived using Fault 1 - Variation in damping, frequency and residual of modes identified by Prony analysis: Model 1, Constant power model; Model 2, 4^{th} order N4SID dynamic equivalent model; Model 3, 5^{th} order N4SID dynamic equivalent model; Model 4, 6^{th} order N4SID dynamic equivalent model; Model 5, 4^{th} order PEM dynamic equivalent model; Model 6, 5^{th} order PEM dynamic equivalent model.

- When the influence of the distribution network on the response to the disturbance is not significant, the constant power model may have better performance than dynamic equivalent models. This might be due to the dynamic equivalent model capturing some dynamic response from the transmission network when it was first derived.
- The lower order dynamic equivalents generally have better ability in presenting the correct damping of the interarea modes.
- The dynamic equivalent usually has better performance in presenting the high frequency modes than the constant power model.

8.6.2 Embedded Wind Generation

The previous analyses were made based on the distribution network being embedded with synchronous generators. The implementation of wind generation lead to modifications of the dynamic characteristics of the distribution network. The following discussion is for the case of the distribution network (see Figure 8.28) embedded with DFIGs (listed in Table 8.6).

Bus number	Generator Type	Number of units	Wind speed (m/s)
67431	Ge1500	3	9
67732	Ge1500	18	9
67832	Ge1500	6	9
68731	Ge1500	18	9
69330	Ge1500	18	9
69339	Ge1500	15	9

Table 8.6: Embedded wind generators

Figure 8.29 and Figure 8.30 show the performance of equivalent models to

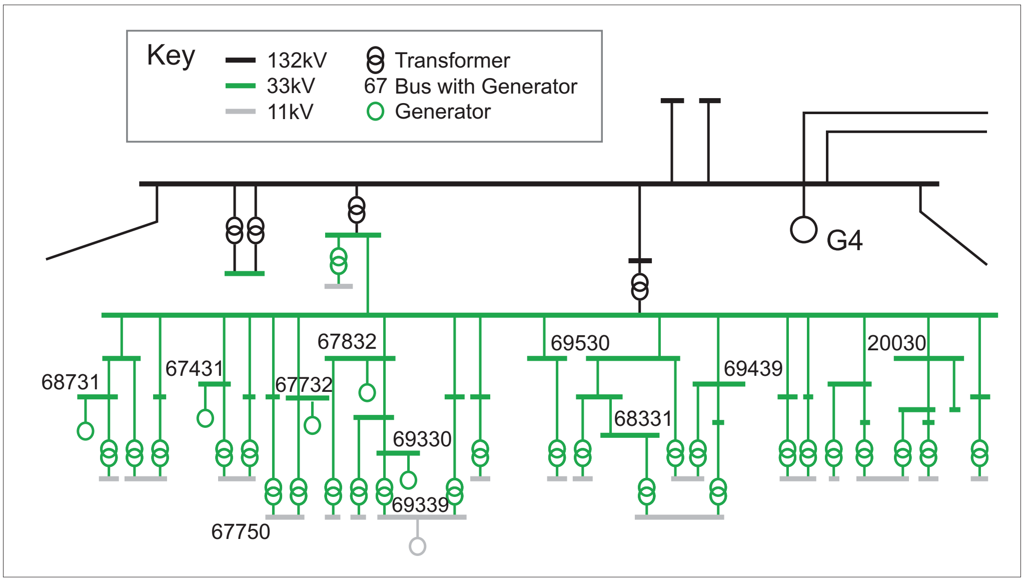
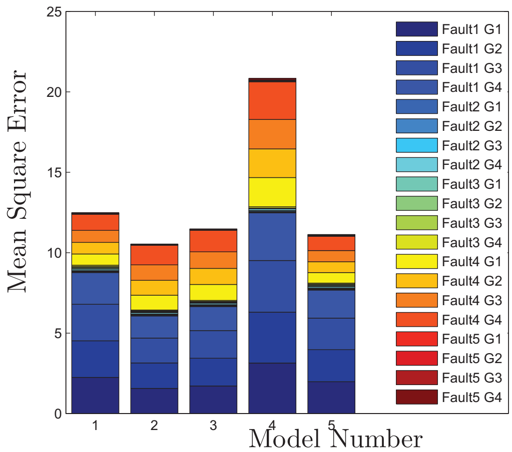
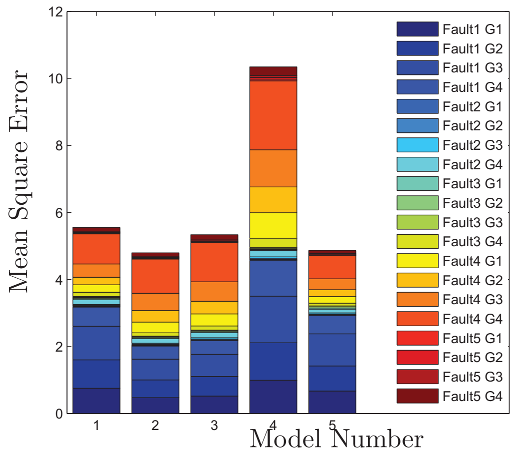


Figure 8.28: Dumfries and Galloway distribution network with embedded DFIGs



(a) Over 9s after the fault



(b) Over 2s after the fault

Figure 8.29: System with embedded wind generators - Performance of dynamic equivalent models derived by Fault 1 in rotor angles of different machines (G1-G4) under different faults (Fault 1-Fault 5): Model 1, Constant power model; Model 2, 4^{th} order N4SID dynamic equivalent model; Model 3, 5^{th} order N4SID dynamic equivalent model; Model 4, 6^{th} order N4SID dynamic equivalent model; Model 5, 4^{th} order PEM dynamic equivalent model.

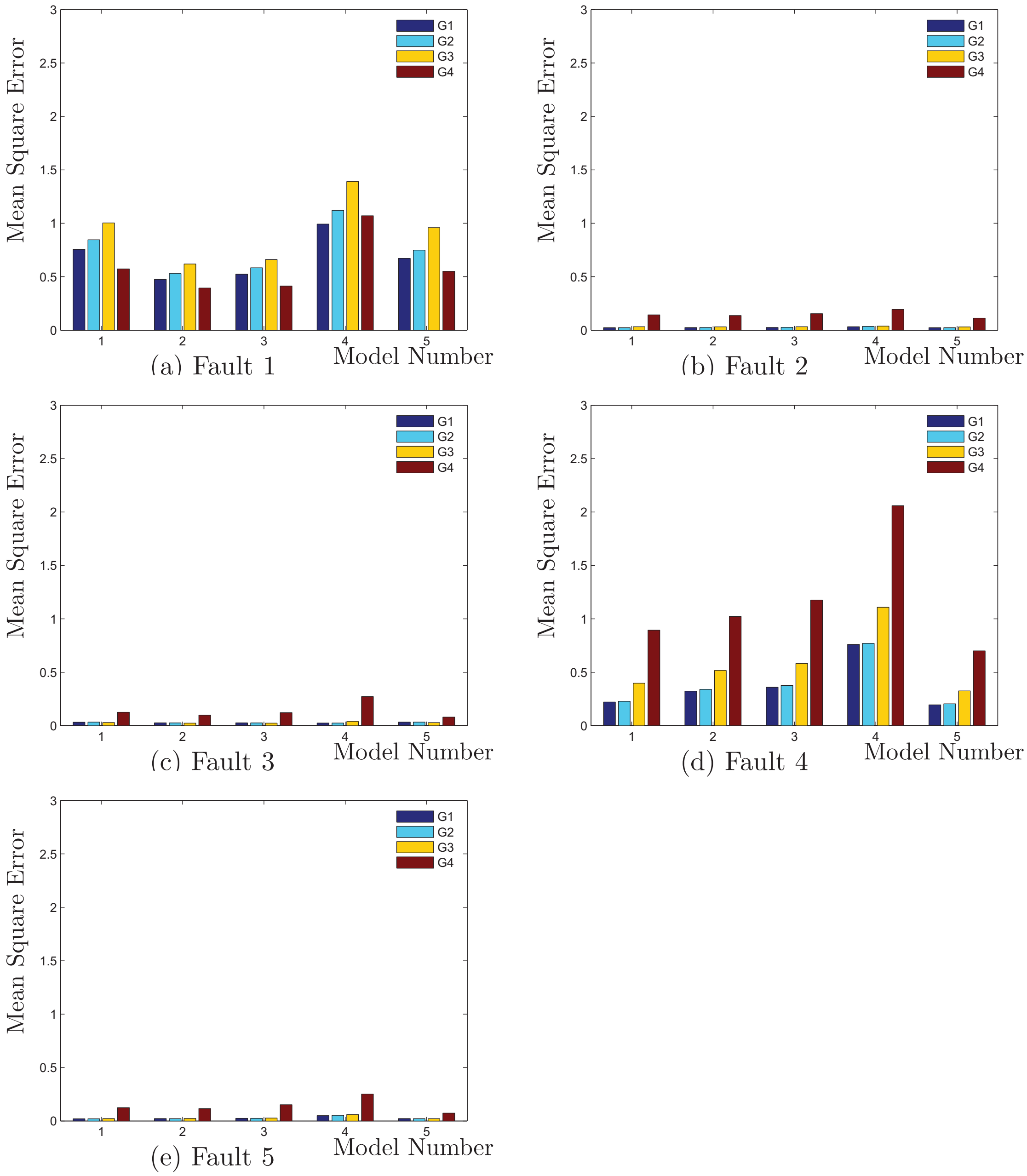


Figure 8.30: System with embedded wind generators - Performance of dynamic equivalent models derived by Fault 1 in rotor angles of different machines (G1-G4) over 2s after different faults: Model 1, Constant power model; Model 2, 4^{th} order N4SID dynamic equivalent model; Model 3, 5^{th} order N4SID dynamic equivalent model; Model 4, 6^{th} order N4SID dynamic equivalent model; Model 5, 4^{th} order PEM dynamic equivalent model.

replace a distribution network embedded with DFIG. From these figures we can see that:

- Unlike the case of the distribution system with embedded synchronous machines, in equivalencing a distribution system with embedded DFIG, the constant power equivalent model has relatively similar performance to the original distribution network model.

  The reason for this is that DFIGs use induction generators. Hence there are synchronous machines in the distribution network to swing coherently against the machines in the transmission network. It therefore has insignificant effects on the interarea modes.

  This can be verified by studying the modes identified by Prony analysis.

- The performance of the responses over 9s or 2s after the fault look similar, which is also different from the synchronous machine case (see Figure 8.3).

- Low order dynamic equivalent models still have better performance than the constant power model in terms of the MSE of machine rotor angles in the transmission network under different faults, either over 9s or 2s after the fault.

Figure 8.32 to Figure 8.38 are identified from the rotor angle responses of the machines in the transmission system (see Figure 8.31). They show the mode position and their damping, frequency and residual errors for the case with embedded wind generators in the distribution system.

Machine G1 Response under Fault 1

From Figure 8.32 and Figure 8.33 we can find out that:

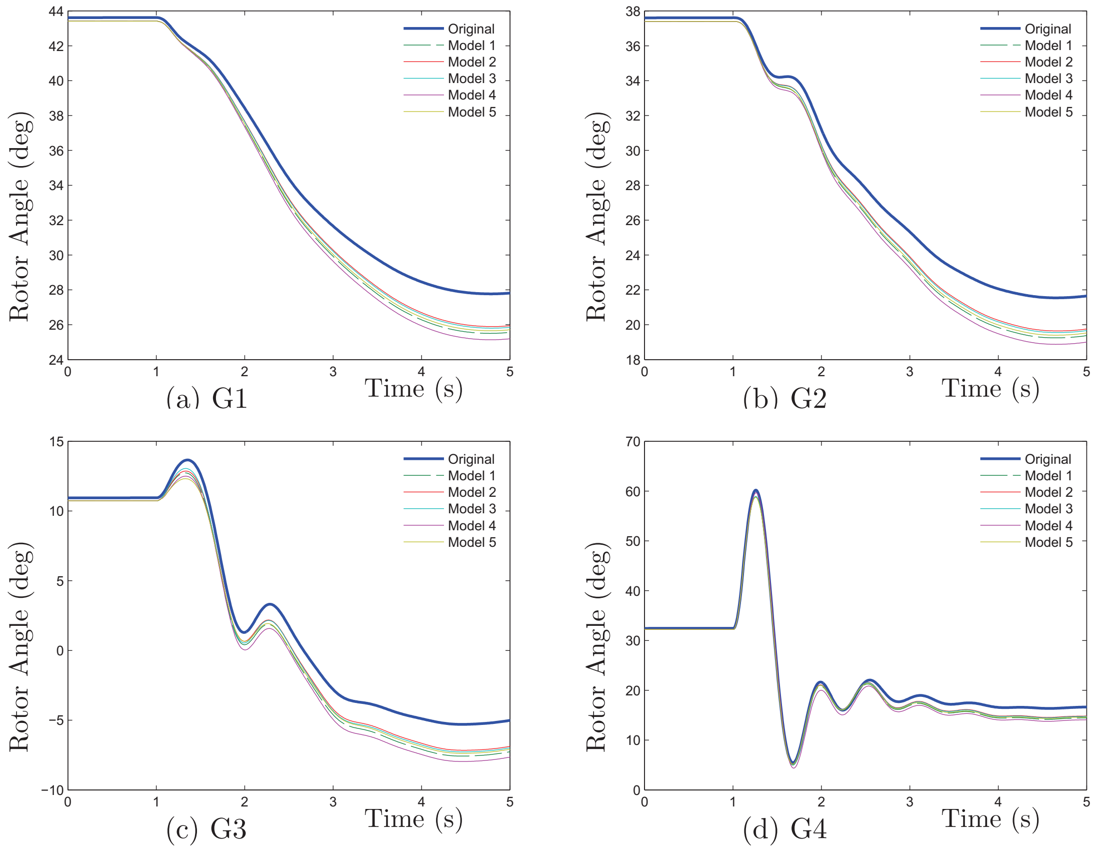


Figure 8.31: System with embedded wind generators - Performance of equivalent models in rotor angles of different machines (G1-G4) under Fault 1: Model 1, Constant power model; Model 2, 4^{th} order N4SID dynamic equivalent model; Model 3, 5^{th} order N4SID dynamic equivalent model; Model 4, 6^{th} order N4SID dynamic equivalent model; Model 5, 4^{th} order PEM dynamic equivalent model.

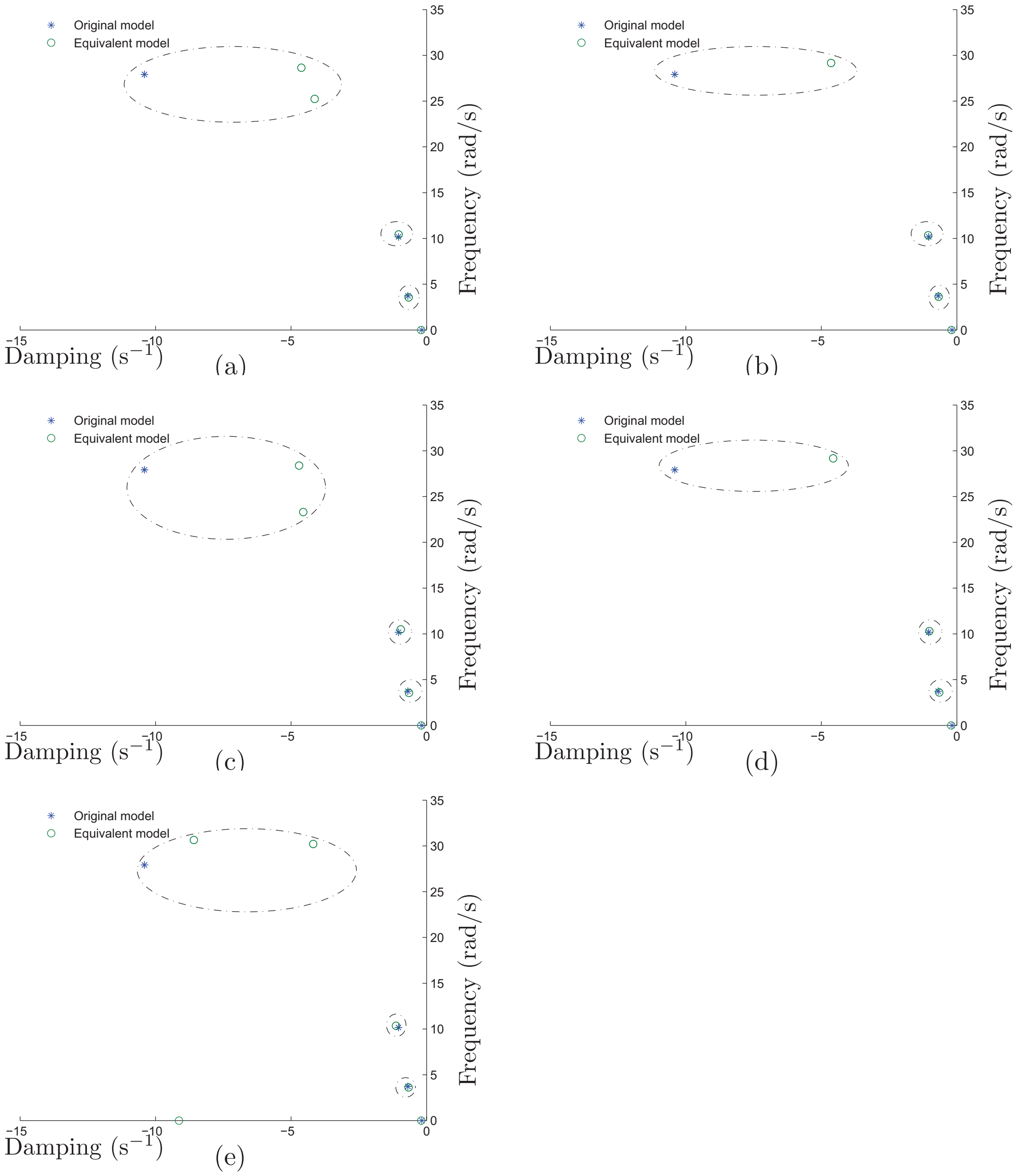
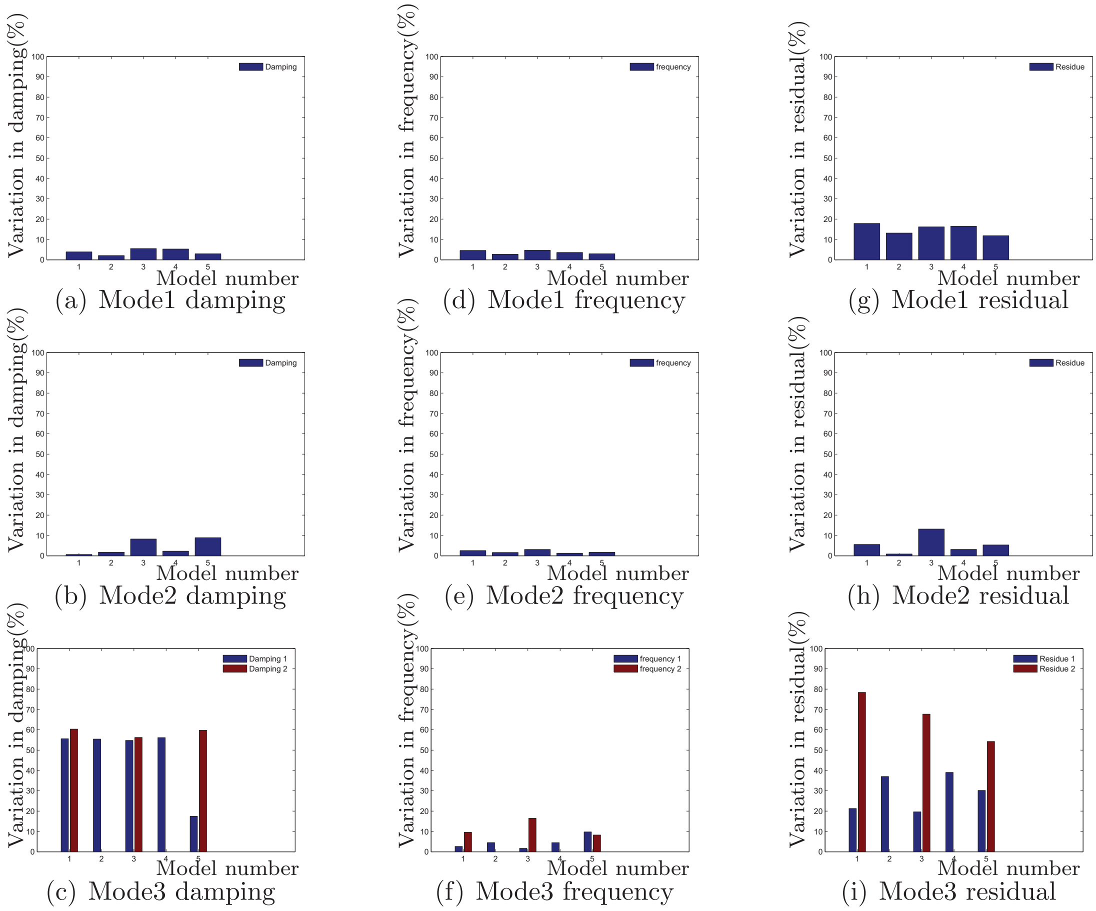


Figure 8.32: System with embedded wind generators - Rotor angle of Machine G1 under Fault 1 when using the equivalent models derived using Fault 1 - Identified modes by Prony analysis: (a) Model 1, Constant power model; (b) Model 2, 4^{th} order N4SID dynamic equivalent model; (c) Model 3, 5^{th} order N4SID dynamic equivalent model; (d) Model 4, 6^{th} order N4SID dynamic equivalent model; (e) Model 5, 4^{th} order PEM dynamic equivalent model.



(a) Mode1 damping (d) Mode1 frequency (g) Mode1 residual

(b) Mode2 damping (e) Mode2 frequency (h) Mode2 residual

(c) Mode3 damping (f) Mode3 frequency (i) Mode3 residual

Figure 8.33: Rotor angle of Machine G1 under Fault 1 when using the equivalent models derived using Fault 1 - Variation in damping, frequency and residual of modes identified by Prony analysis: Model 1, Constant power model; Model 2, 4^{th} order N4SID dynamic equivalent model; Model 3, 5^{th} order N4SID dynamic equivalent model; Model 4, 6^{th} order N4SID dynamic equivalent model; Model 5, 4^{th} order PEM dynamic equivalent model.

- Interarea modes

  In theory, the major difference between the constant power model and the original distribution network model should be in the interarea modes. According to the frequency level, the first circle from the real axis marks the interarea modes (mode 1) in the figure.

  As we can see, unlike the case of distribution network with synchronous generators, the damping difference for the constant power model is insignificant. Not much shift can be seen in the mode position.

  As shown in Figure 8.33, all the equivalent models (Model 1-6) have similar performance in matching the interarea modes well, with no shift, which could be seen in synchronous EG case. The $4th$ order dynamic equivalent model (Model 2) has better performance in presenting the interarea mode than the constant power model (Model 1) although the advantage is modest.

- Machine modes

  The second circle marks the machine modes (mode 2) and all the equivalent models have good performance in representing the machine modes.

- High frequency modes

  All the equivalent models have poor performance in representing the high frequency modes.

Machine G2 Response under Fault 1

Figure 8.34 and Figure 8.35 have shown that:

- In Figure 8.34, machine mode and interarea mode of power system were split as two modes by equivalent models.

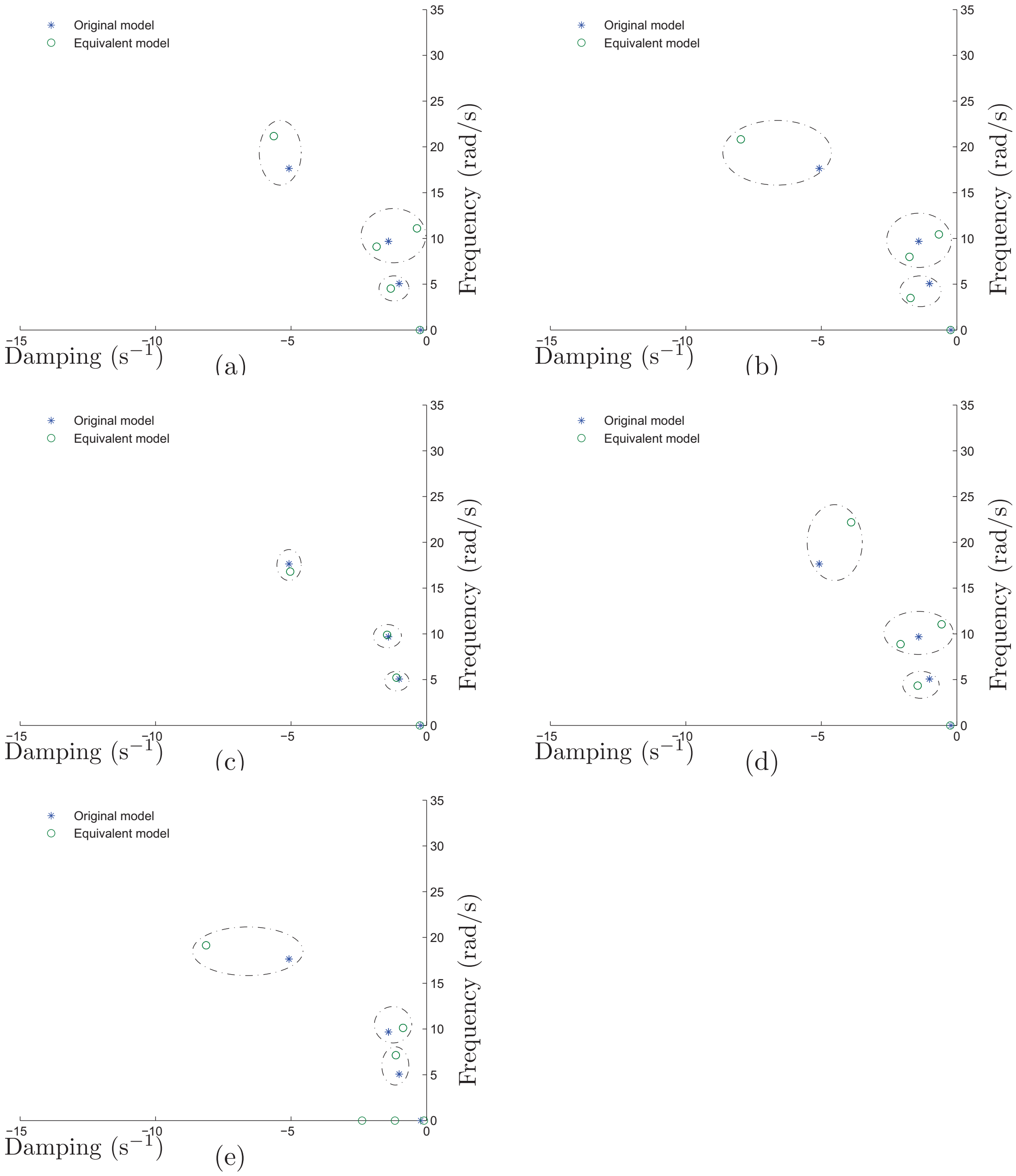
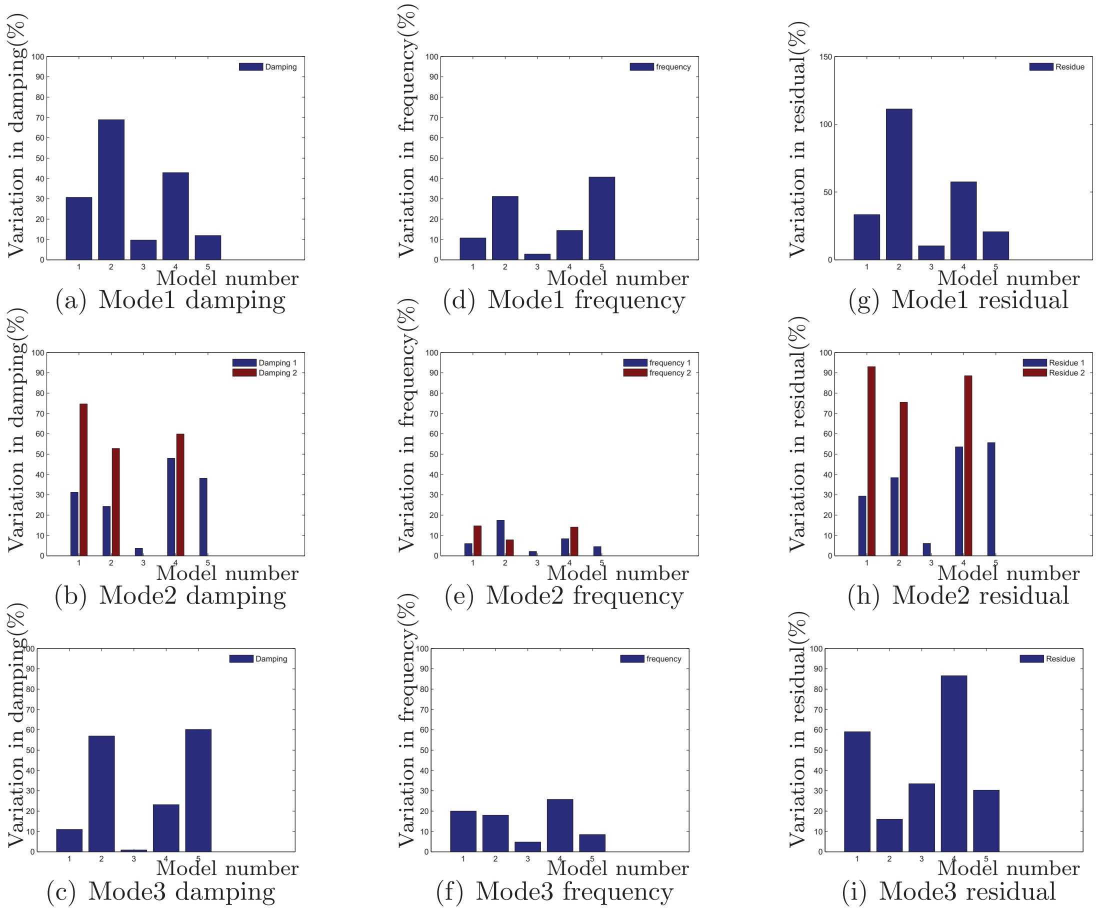


Figure 8.34: System with embedded wind generators - Rotor angle of Machine G2 under Fault 1 when using the equivalent models derived using Fault 1 - Identified modes by Prony analysis: (a) Model 1, Constant power model; (b) Model 2, 4^{th} order N4SID dynamic equivalent model; (c) Model 3, 5^{th} order N4SID dynamic equivalent model; (d) Model 4, 6^{th} order N4SID dynamic equivalent model; (e) Model 5, 4^{th} order PEM dynamic equivalent model.



(a) Mode1 damping (d) Mode1 frequency (g) Mode1 residual

(b) Mode2 damping (e) Mode2 frequency (h) Mode2 residual

(c) Mode3 damping (f) Mode3 frequency (i) Mode3 residual

Figure 8.35: Rotor angle of Machine G2 under Fault 1 when using the equivalent models derived using Fault 1 - Variation in damping, frequency and residual of modes identified by Prony analysis: Model 1, Constant power model; Model 2, 4^{th} order N4SID dynamic equivalent model; Model 3, 5^{th} order N4SID dynamic equivalent model; Model 4, 6^{th} order N4SID dynamic equivalent model; Model 5, 4^{th} order PEM dynamic equivalent model.

- The $5th$ order dynamic equivalent model (Model 3) has the best performance in presenting all the modes.

Machine G3 Response under Fault 1

From Figure 8.36 and Figure 8.37 we can draw a conclusion that unlike what happened with the synchronous case where lower order dynamic equivalents can represent the interarea modes better, the constant power model has overall the best performance in presenting all the modes, including the high frequency modes.

Machine G4 Response under Fault 1

Figure 8.38 and Figure 8.39 have shown that all the equivalent models have very similar performance in representing the modes.

Conclusion

In the case of the distribution network embedded with synchronous generators, the major difference between the constant power and dynamic equivalent was the interarea modes. However in the case of the distribution network embedded with wind generators, which is usually based on induction generators, there are no groups of synchronous machines in the distribution network to swing coherently against the machines in the transmission network (i.e. there is no interarea modes).

Hence in any of the machine responses, the identified interarea modes have no obvious difference between the constant power and the dynamic equivalent models. This is quite different from the embedded synchronous generator case under the same fault.

Therefore, the advantage of the dynamic equivalencing for distribution

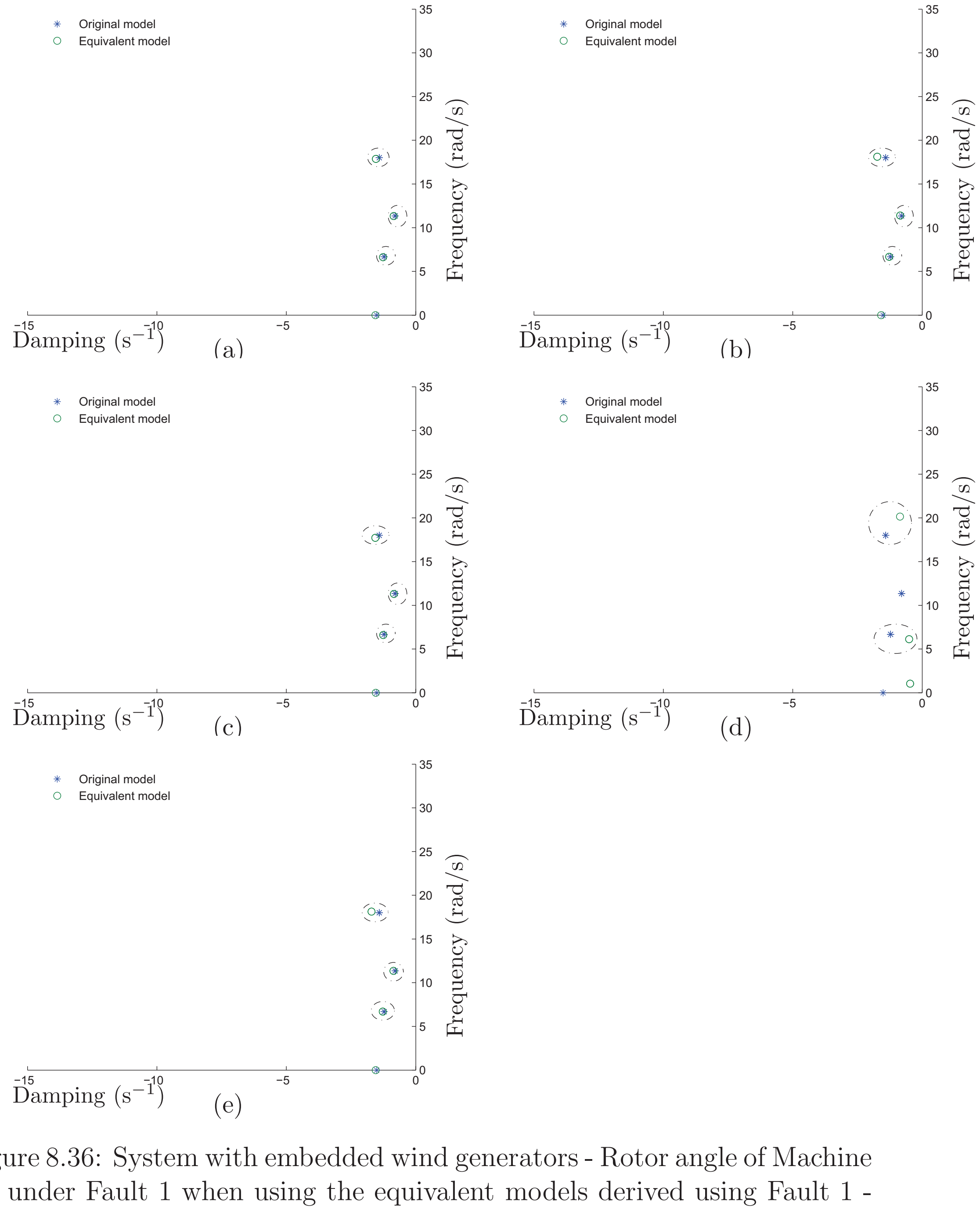


Figure 8.36: System with embedded wind generators - Rotor angle of Machine G3 under Fault 1 when using the equivalent models derived using Fault 1 - Identified modes by Prony analysis: (a) Model 1, Constant power model; (b) Model 2, 4^{th} order N4SID dynamic equivalent model; (c) Model 3, 5^{th} order N4SID dynamic equivalent model; (d) Model 4, 6^{th} order N4SID dynamic equivalent model; (e) Model 5, 4^{th} order PEM dynamic equivalent model.

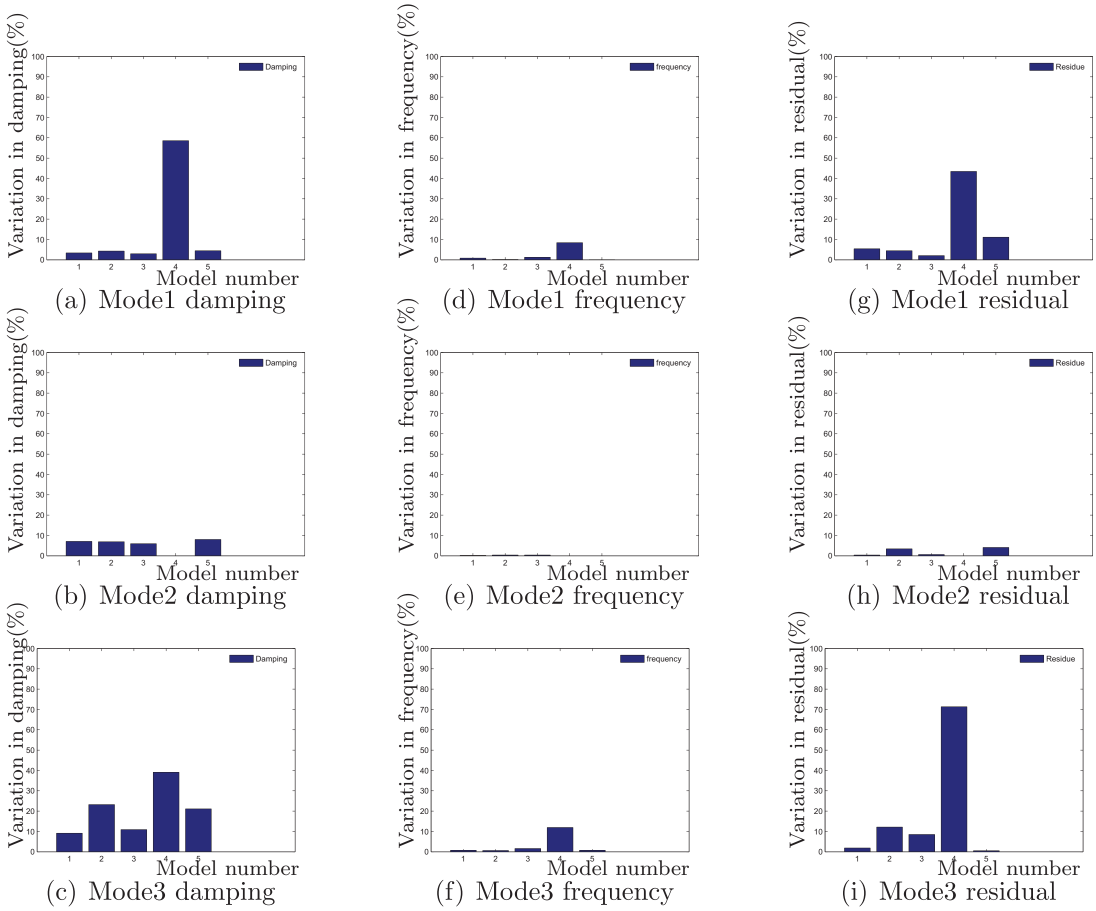


Figure 8.37: Rotor angle of Machine G3 under Fault 1 when using the equivalent models derived using Fault 1 - Variation in damping, frequency and residual of modes identified by Prony analysis: Model 1, Constant power model; Model 2, 4^{th} order N4SID dynamic equivalent model; Model 3, 5^{th} order N4SID dynamic equivalent model; Model 4, 6^{th} order N4SID dynamic equivalent model; Model 5, 4^{th} order PEM dynamic equivalent model

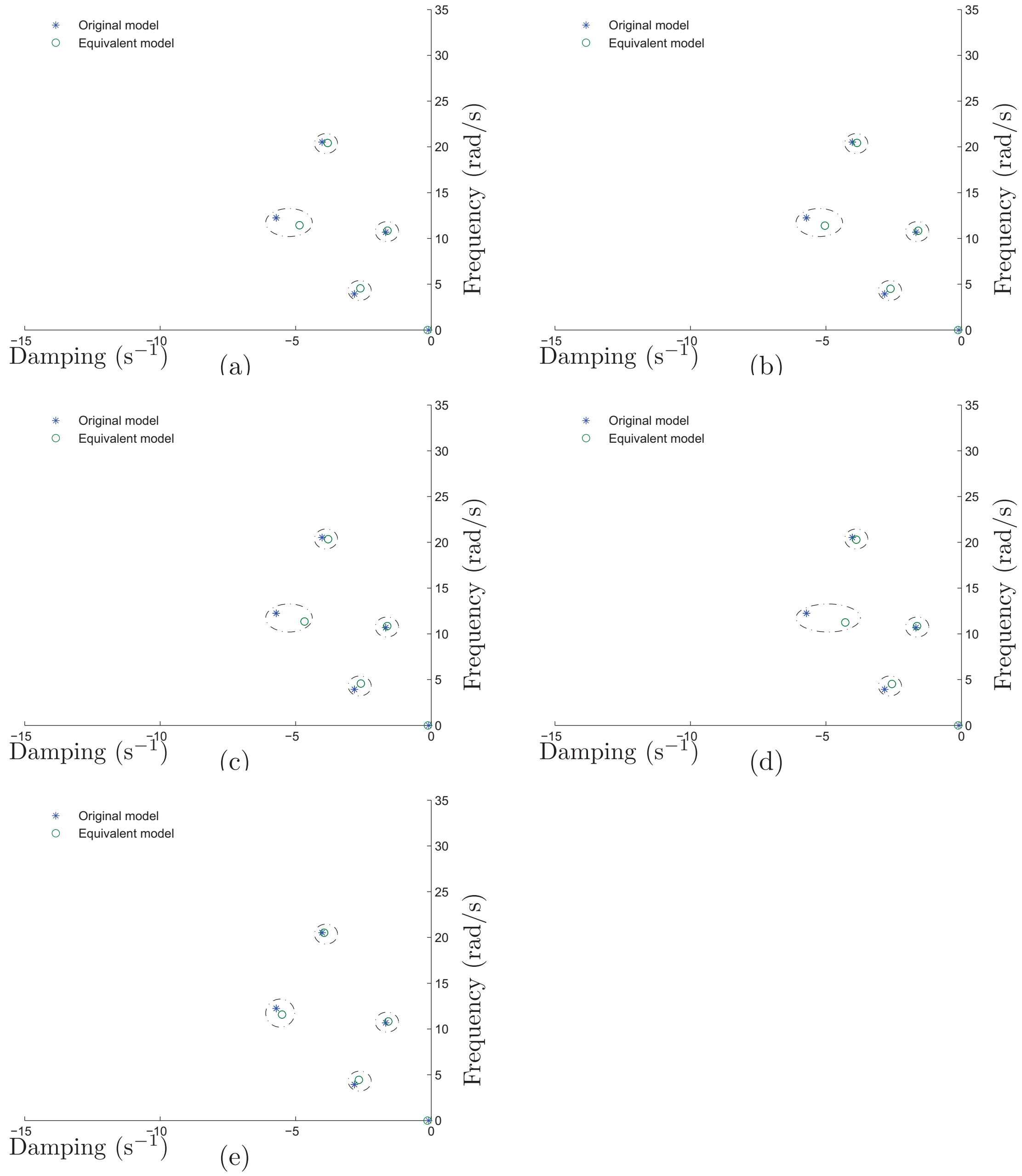


Figure 8.38: System with embedded wind generators - Rotor angle of Machine G4 under Fault 1 when using the equivalent models derived using Fault 1 - Identified modes by Prony analysis: (a) Model 1, Constant power model; (b) Model 2, 4^{th} order N4SID dynamic equivalent model; (c) Model 3, 5^{th} order N4SID dynamic equivalent model; (d) Model 4, 6^{th} order N4SID dynamic equivalent model; (e) Model 5, 4^{th} order PEM dynamic equivalent model.

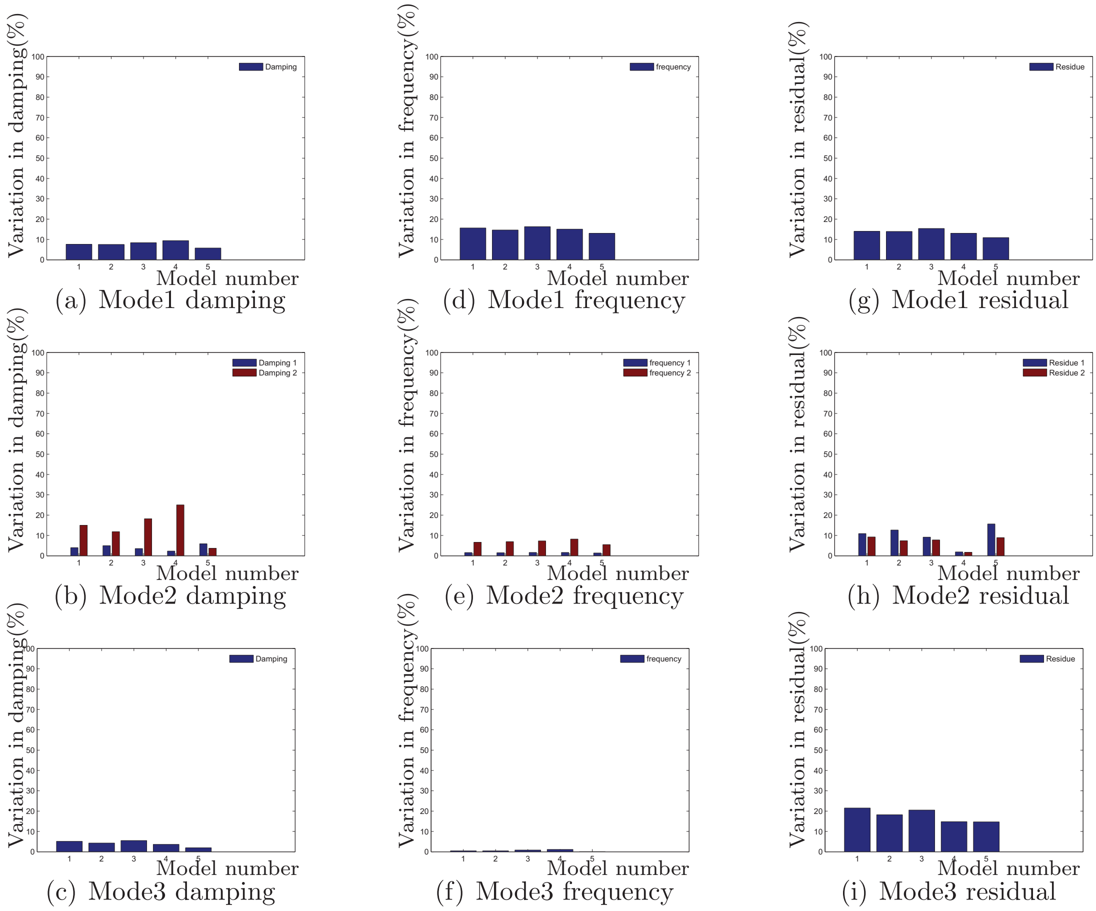


Figure 8.39: Rotor angle of Machine G4 under Fault 1 when using the equivalent models derived using Fault 1 - Variation in damping, frequency and residual of modes identified by Prony analysis: Model 1, Constant power model; Model 2, 4^{th} order N4SID dynamic equivalent model; Model 3, 5^{th} order N4SID dynamic equivalent model; Model 4, 6^{th} order N4SID dynamic equivalent model; Model 5, 4^{th} order PEM dynamic equivalent model.

network with embedded wind generators is not very obvious compared with the case with embedded synchronous generators. This can be seen clearly in the MSE values.

8.7 Conclusion

This chapter evaluates the performance of different equivalent models by comparing their performance with that of the original distribution network model under the same operating conditions. State-space dynamic equivalent models of different orders and a constant power equivalent model have been tested in dynamic simulation and analysed based on the measures introduced in the previous chapter.

Factors that influence the performance of the equivalents are discussed and some conclusions can be drawn from the analysis in this chapter:

- *The dynamic equivalent should be derived using single-input data instead of double-input data.* The performance of the dynamic equivalent models derived from the two inputs, V and f, are worse than those derived from the V input only. This is because the frequency f at the connecting bus may change very insignificantly in the original power system. The lack of richness of the signal may derive an inaccurate equivalent model, whilst a small variation in f can lead to a considerable change in the model output. These affect the model performance and even stimulate instability in the power system.

- *A low order (i.e. 4th order) state-space model derived using N4SID algorithm is suggested for use as the dynamic equivalent model of a distribution network.* The N4SID model is in general as good as, if not better than, the PEM model. For dynamic equivalent models derived using either N4SID or PEM algorithms, the $4th$ order dynamic equivalent models may always behave better than higher orders. A possible reason for this might be that the higher order models may include more effects from the transmission network side than the lower order ones.

- *A line fault close to the distribution network was proved to be an appropriate disturbance for deriving the dynamic equivalent of distribution network.* The richness of the signal determines the accuracy of the dynamic equivalent models derived from it. A short-circuit fault is a good choice since it causes relatively large oscillations of voltage at the connecting bus. The disturbance close to the distribution network avoids involving much transmission network response in dynamic equivalencing of the distribution network.

- *A small distribution network can be equivalenced as a constant power model.* With the distribution network size increasing, the advantage of the dynamic equivalent model over the constant power equivalent model will become more obvious. However, if the distribution network is very small, the constant power equivalent model may have better performance than dynamic equivalent models.

- *For distribution system embedded with DFIG wind generation, the dynamic equivalent models have no significant advantage over the constant power model.* For the distribution network embedded with synchronous generators, the dynamic equivalents have apparently better performance than the constant power model. However, when the distribution network is embedded with wind generators, the advantage is not obvious, as the distribution network embedded with only induction generators may have no interarea modes. In this case, the constant power model can have similar performance to the dynamic equivalent models.

These discussions are helpful for selecting proper model structure and disturbances to derive the equivalent models, and also give reasonable suggestions on the application of this dynamic equivalencing approach.

Chapter 9

Conclusions and Future Work

9.1 Thesis Summary

Because of the historically insignificant contribution of embedded generation to the power system, traditionally EG has been treated by the transmission system operator as negative load, with its impact on dynamic behaviour of power system neglected.

With the increasing penetration level of EG caused by development of renewable generation implemented in the distribution network, EG may start to influence the dynamic behavior and the stability problem of the transmission network. This change requires analysis of the dynamics and stability of the distribution network.

As the detailed data of the distribution network is not always available to the transmission system operator, a dynamic equivalent model of the distribution network is required.

Most dynamic equivalencing methods are based on the assumption that detailed information of the network is known. This is not the case for the distribution network. Some dynamic equivalencing methods based on coherency of machines, which have been applied to the transmission network cannot be applied to the distribution network due to its radial structure. Hence a simulation-based dynamic equivalencing methodology has been developed in this project, which could derive the dynamic equivalent model of the distribution network using system identification, without detailed information on the distribution network necessarily known.

A case study has been accomplished in PSS/E on a model of the Scottish transmission network with a distribution network in Dumfries and Galloway. Embedded generation with a certain penetration level in either conventional generation or DFIG wind generation has been added to the model of the distribution network. Three-phase faults located at different places in the transmission network have been used as a disturbance to derive the dynamic equivalent of distribution network with EG. After being built by system identification from simulations, these dynamic equivalent models have been implemented in PSS/E as a user-defined model. Their performance is compared with the original distribution network using analytical indicators. A constant power model has also been involved for comparison to illustrate the advantage of using the dynamic equivalent to represent the distribution network.

Finally, factors that may affect the quality of the model during the dynamic equivalencing procedure have been discussed. Advice has been given for each step of this dynamic equivalencing methodology, together with the Fortran code used for the PSS/E user-defined model implementation listed in the Appendix.

9.2 Conclusion

Large amount of EG units being implemented in the distribution network can significantly affect the system dynamic behaviour and all types of stability: The fluctuating nature of renewable sources and their sensitive protection devices may affect the frequency stability of the power system; The renewable generators may have less ability to control the grid voltage than conventional generators; The voltage stability problems may be caused by misuse of the control system for EG; Furthermore, EG units may behave differently from conventional generators and impact on power system transient stability.

The existing dynamic equivalencing approaches can be divided into two main categories: modal analysis and coherency-based approach. With the increasing complexity of the power system and the new components and customised control devices implemented in the system, the application of these dynamic equivalencing approaches may become limited as they require the detailed information of the power system in advance.

The dynamic equivalencing methodology developed in this project is based on system identification. While the methodology is intended for use with real measurements, by necessity simulation results from detailed simulations are used.

The derived dynamic equivalents and a constant power equivalent have been compared in representing the original distribution network response. Along with visually analysing the dynamic response of the system, analytical indicators including mean square error, cross-correlation sequence peak values and Prony mode positions, are used for evaluating the model performance. All these indicators suggest the better agreement of the dynamic equivalent responses to those of the original distribution system than the constant power equivalent.

Some conclusions are offered on the factors that influence the performance of the dynamic equivalents.

The dynamic equivalent should be derived using voltage. The model derived using voltage together with frequency provides very poor performance in simulations.

A state-space model is suggested to be selected. Two popular identification algorithms, N4SID and PEM, for the state-space model are explained and compared. Simulation results indicate the N4SID model is in general as good as, if not better than, the PEM model.

The proper order of the dynamic equivalent can be determined by the singular value diagram which can reflect the order of the distribution network. Simulations have suggested that this or a slightly higher order is enough for the state-space model to represent the distribution network. For models derived using either N4SID or PEM algorithms, the $4th$ order (determined by the singular value diagram) models always behave better than higher order ones.

A line fault close to the distribution network was proved to be an appropriate disturbance for deriving the dynamic equivalent of distribution network. The richness of the signal determines the accuracy of the dynamic equivalent models derived using it. A short-circuit fault is a good choice since it causes relatively large oscillations of voltage at the connecting bus. The disturbance close to the distribution network avoids involving much transmission network response in dynamic equivalencing of the distribution network.

A small distribution network can be equivalenced as a constant power model. If the distribution network is very small, the constant power model may behave better than the dynamic equivalent models. With the distribution network size increasing, the advantage of the dynamic equivalent model over the constant power equivalent model will become more obvious.

For the distribution network embedded with synchronous generators, the dynamic equivalents have better performance than the constant power model. However, for distribution systems embedded totally with DFIGs, the dynamic equivalent models have no significant advantage over the constant power model as induction generators may have no inter-area modes. In this case, the constant power model can give similar performance to the dynamic equivalent models.

9.3 Future Work

The major advantage of this dynamic equivalencing methodology is that it uses the time series obtained from the simulation to derive the dynamic equivalent models. A further advantage is that the approach lends itself to direct measurement in the real power system. Future work could possibly concentrate on the application of this dynamic equivalencing methodology in real networks.

The limitation of this methodology is that the accuracy of the model is not perfect. More system identification strategies and disturbance selections could be evaluated in their ability to derive good dynamic equivalents. Studies on how to isolate the impact of transmission network on distribution network during the dynamic equivalencing would be particularly valuable.

Also, the current work that has been done is about deriving dynamic equivalent of a distribution network with EGs. The application of this methodology for transmission level network analysis could also be an area for studies.

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Appendix

A N4SID Identification Algorithm

State-space model can be written in the innovations form, which describes noise [107]:

$$
\begin{aligned}
x(k+1) &= Ax(k) + Bu(k) + Ke(k) && (1)\\
y(k) &= Cx(k) + Du(k) + e(k) && (2)
\end{aligned}
$$

The basic idea is to split the general form model into two subsystems: a deterministic subsystem (denoted as d) and a stochastic subsystem (denoted as s). Then we have the states and outputs described as [110]:

$$x_k = x_k^d + x_k^s \tag{3}$$
$$y_k = y_k^d + y_k^s \tag{4}$$

- The deterministic subsystem

$$
\begin{aligned}
x_{k+1}^d &= Ax_k^d + Bu_k && (5)\\
y_k^d &= Cx_k^d + Du_k && (6)
\end{aligned}
$$

This subsystem is defined by the evolution of the system dynamics, which describes the influence of the deterministic input u_k on the deterministic output y_k^d. Associated with the deterministic subsystem, we define the extended observability matrix Γ_i as:

$$\Gamma_i = \begin{pmatrix} C \\ CA \\ CA^2 \\ \vdots \\ CA^{i-1} \end{pmatrix} \tag{7}$$

The reversed extended deterministic controllability matrix Δ_i^d is:

$$\Delta_i^d = [A^{i-1}B \quad A^{i-2}B \quad \cdots \quad AB \quad B] \tag{8}$$

The deterministic lower block triangular Toeplitz matrix H_i^d is given by:

$$H_i^d = \begin{pmatrix} D & 0 & 0 & \cdots & 0 \\ CB & D & 0 & \cdots & 0 \\ CAB & CB & D & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots & 0 \\ CA^{i-2}B & CA^{i-3}B & CA^{i-4}B & \cdots & D \end{pmatrix} \tag{9}$$

- The stochastic subsystem

$$x_{k+1}^s = Ax_k^s + Ke_k \tag{10}$$
$$y_k^s = Cx_k^s + e_k \tag{11}$$

This subsystem is purely driven by noise, which describes the noise sequence Ke_k and e_k on the stochastic output y_k^s.

The stochastic lower block Finally triangular Toeplitz matrix H_i^s is given by:

$$H_i^s = \begin{pmatrix} 0 & 0 & 0 & \cdots & 0 \\ C & 0 & 0 & \cdots & 0 \\ CA & C & 0 & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots & 0 \\ CA^{i-2} & CA^{i-3} & CA^{i-4} & \cdots & 0 \end{pmatrix} \tag{12}$$

- Block Hankel matrices and input output equations
  The input and output block Hankel matrices are defined as:

$$U_{0|i-1} = \begin{pmatrix} u[0] & u[1] & \cdots & u[j-1] \\ u[1] & u[2] & \cdots & u[j] \\ \vdots & \vdots & \vdots & \vdots \\ u[i-1] & u[i] & \cdots & u[i+j-2] \end{pmatrix} \tag{13}$$

$$Y_{0|i-1} = \begin{pmatrix} y[0] & y[1] & \cdots & y[j-1] \\ y[1] & y[2] & \cdots & y[j] \\ \vdots & \vdots & \vdots & \vdots \\ y[i-1] & y[i] & \cdots & y[i+j-2] \end{pmatrix} \tag{14}$$

For convenience and short notation:

$$U_p = U_{0|i} \tag{15}$$
$$U_f = U_{i|2i-1} \tag{16}$$
$$Y_p = Y_{0|i} \tag{17}$$
$$Y_f = Y_{i|2i-1} \tag{18}$$

Where the subscript p and f, denote respectively the past and the future.

Using this matrix notation, the system equations are as follows:

$$Y_p = \Gamma_i X_p^d + H_i^d U_p + Y_p^s \tag{19}$$
$$Y_f = \Gamma_i X_f^d + H_i^d U_f + Y_f^s \tag{20}$$
$$Y_p^s = \Gamma_i X_p^s + H_i^s M_p + N_p \tag{21}$$
$$Y_f^s = \Gamma_i X_f^s + H_i^s M_f + N_f \tag{22}$$
$$X_f^d = A^i X_p^d + \Delta_i^d U_p \tag{23}$$

Where
Y_p^s, Y_f^s are the block Hankel matrix formed with the outputs y_k^s of the stochastic subsystem.
M_p, M_f are the block Hankel matrix formed with the process noise Ke_k.
N_p, N_f are the block Hankel matrix formed with the measurement noise e_k.
The past and future deterministic and stochastic state sequences are:

$$\begin{array}{l} X_p^d = [x^d[0] \quad x^d[1] \quad \cdots \quad x^d[j-1]] \\ X_f^d = [x^d[i] \quad x^d[i+1] \quad \cdots \quad x^d[i+j-1]] \\ X_p^s = [x^s[0] \quad x^s[1] \quad \cdots \quad x^s[j-1]] \\ X_f^s = [x^s[i] \quad x^s[i+1] \quad \cdots \quad x^s[i+j-1]] \end{array} \tag{24}$$

The weighted projection O_i can be written as the oblique projection of the future outputs Y_f into the past input and output space W_p along the future input U_f:

$$O_i = W_1 \cdot Y_f /_{U_f} W_p = W_1 \cdot \Gamma_i \tilde{X}_f \tag{25}$$

Where Γ_i is the observability matrix and $\tilde{X}_f$ is an estimated state sequence of X_f.

Under the conditions of excitation of all the modes of interest, the observability matrix Γ_i and an estimated $\tilde{X}_f$ of the state sequence can be recovered from:

$$\Gamma_i = W_{-1} \cdot U_1 S_1^{1/2} \tag{26}$$
$$\tilde{X}_f = S_1^{1/2} V_1^T \tag{27}$$

With the obvious definition of white noise, after the state sequence has been estimated, a least square procedure can be used to estimate the system matrices from the matrix equation [108]:

$$min_{A,B,C,D} \left\| \begin{bmatrix} \tilde{X}_{i+1} \\ Y_{i|i} \end{bmatrix} - \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} \tilde{X}_i \\ U_{i|i} \end{bmatrix} \right\| \tag{28}$$

B Prony Analysis

$F(s)$, the Laplace transform of a function f(t) can be written in the form of:

$$F(s) = \frac{r_1}{s-p_1} + \frac{r_2}{s-p_2} + \cdots + \frac{r_n}{s-p_n} \tag{29}$$

p_k: pole ($k = 1, 2, \cdots, n$)
r_k: residue of p_k

$$\begin{aligned} f(t) &= \mathcal{L}^{-1}[F(s)] && (30) \\ &= r_1 e^{p_1 t} + r_2 e^{p_2 t} + \cdots + r_n e^{p_n t} && (31) \\ &= \sum_{i=1}^{n} r_i e^{p_i t} && (32) \end{aligned}$$

Let $f(t)$ be a discrete signal consisting of N evenly spaced samples:

$$f(k) = f(t_k),\ (k = 0, 1, \cdots, N-1) \tag{33}$$

Prony's method can directly estimate the poles by fitting a sum of complex damped sinusoids to $f(t)$:

$$\hat{f}(t) = \sum_{i=1}^{Q} A_i e^{\sigma_i t} cos(2\pi f_i t + \phi_i) \tag{34}$$

$\hat{f}(t)$: Estimate of the observed $f(t)$
A_i: Amplitute of component i
σ_i: Damping coefficeite of component i
ϕ_i: Phase of component i
f_i: Frequency of component i

According to Euler's formula,

$$cos\theta = \frac{1}{2}(e^{j\theta} + e^{-j\theta}) \tag{35}$$

function 34 can be written in a simplified form :

$$\begin{aligned} \hat{f}(t) &= \sum_{i=1}^{Q} (\frac{1}{2} A_i e^{\sigma_i t} e^{j2\pi f_i t + \phi_i} + \frac{1}{2} A_i e^{\sigma_i t} e^{-j2\pi f_i t - \phi_i}) && (36) \\ &= \sum_{i=1}^{Q} (\frac{1}{2} A_i e^{\phi_i} e^{(\sigma_i + j2\pi f_i)t} + \frac{1}{2} A_i e^{-\phi_i} e^{(\sigma_i - j2\pi f_i)t}) && (37) \\ &= \sum_{i=1}^{P} B_i e^{\lambda_i t} && (38) \end{aligned}$$

Where
$B_i = \frac{1}{2} A_i e^{\phi_i}$ or $\frac{1}{2} A_i e^{-\phi_i}$
$\lambda_i = \sigma_i + j2\pi f_i$ or $\sigma_i - j2\pi f_i$

Letting $t = kT$, in parallel with $f(t)$, the samples of $\hat{f}(t)$ are written as:

$$\hat{f}(k) = \sum_{i=1}^{P} B_i \mu_i^k,\ (k = 0, 1, \cdots, N-1) \tag{39}$$

$$\mu_i = e^{\lambda_i T} \tag{40}$$

μ_i: Poles
T: Sampling time period

The objective is to find the B_i and μ_i that produces $\hat{f}(k) = f(k)$

$$\begin{bmatrix} \mu_1^0 & \mu_2^0 & \cdots & \mu_n^0 \\ \mu_1^1 & \mu_2^1 & \cdots & \mu_n^1 \\ \vdots & & & \\ \mu_1^{N-1} & \mu_2^{N-1} & \cdots & \mu_n^{N-1} \end{bmatrix} \begin{bmatrix} B_1 \\ B_2 \\ \vdots \\ B_n \end{bmatrix} = \begin{bmatrix} f(0) \\ f(2) \\ \vdots \\ f(N-1) \end{bmatrix} \tag{41}$$

This can be done by solving the linear prediction model:

$$f(k) = a_1 f(k-1) + a_2 f(k-2) + \cdots + a_n f(k-n) \tag{42}$$

for $k = n, n+1, n+2, \cdots, N-1$

$$\begin{bmatrix} f(n) \\ f(n+1) \\ \vdots \\ f(N-1) \end{bmatrix} = \begin{bmatrix} f(n-1) & f(n-2) & \cdots & f(0) \\ f(n) & f(n-1) & \cdots & f(1) \\ \vdots & & & \\ f(N-2) & f(N-3) & \cdots & f(N-n-1) \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_n \end{bmatrix} \tag{43}$$

The coefficient vector a can be calculated. Eigenvalues $\hat{\mu_i}$ are obtained by calculating the roots of the characteristic polynomial formed from the linear prediction coefficients.

$$\mu^n - a_1\mu^{n-1} - \cdots - a_{n-1}\mu - a_n = (\mu - \hat{\mu_1})(\mu - \hat{\mu_2})\cdots(\mu - \hat{\mu_n}) \tag{44}$$

As $\hat{\mu_i}$ is known, B_i can be calculated by solving the equation (41). The amplitude, frequency, phase and damping coefficient are computed using equation (38).

Prony analysis can be used to identify the damping and frequency of the modes from a signal. It should be noticed that the number of the modes needs to be determined in advance. Usually this number is less than the number of the real modes and is associated with sample number of the data series used for identification. Hence each identified Prony mode may represent more than one real modes of the system.

C Python Code

A Python code is written for compiling the model description generated by Matlab System Identification Toolbox to Fortran code, which could be recognised by PSS/E user-defined model. Files that contain matrices should be put under the same directory to the Python code file. Compilation could be done in batch.

```
def EqImport(MinNum,MaxNum,FileName): #general function definition
input=open('C:\\EqABC\\f\\TD5O_.f','r')
```

```
Top=input.read()
input=open(’C:\\EqABC\\f\\_TD5O0.f’,’r’)
Bottom=input.read()
for Num in range(MinNum,MaxNum):
TxtFile=str(FileName)+str(Num+1)+’.txt’
#print SaveFileNo
input=open(TxtFile,’r’) #creat input file(’r’ means read)
S=input.read()
for i in range(1,6):
S=S.replace(’x’+str(i),”)
X=’=ABCPQV’
for item in X:
S=S.replace(item,”)
S=S.split()
A=’ A = [’
for i in range(0,5):
A=A+S[i]+’, ’
A=A+’\n &’
for i in range(5,10):
A=A+S[i]+’, ’
A=A+’\n &’
for i in range(10,15):
A=A+S[i]+’, ’
A=A+’\n &’
for i in range(15,20):
A=A+S[i]+’, ’
A=A+’\n &’
for i in range(20,24):
A=A+S[i]+’, ’
A=A+S[24]+’]\n’
B=’ B = [’
for i in range(25,29):
B=B+S[i]+’, ’
B=B+S[29]+’]\n’
C=’ C = [’
for i in range(30,35):
C=C+S[i]+’, ’
C=C+’\n &’
for i in range(35,39):
C=C+S[i]+’, ’
C=C+S[39]+’]\n’
input.close()
#print A B C
output=open(’5_’+str(Num+1)+’.flx’,’w’)
output.write(Top)
output.write(A)
output.write(B)
output.write(C)
output.write(Bottom)
```

```
output.close()
#end of function definition and begin of the main module
Spath='C:\\EqABC\\'
EqImport(0,1,'x')
```

D Fortran Code

Double Inputs Model

```
SUBROUTINE EQUIBL(I,ISLOT,ISLOT2)
C
$INSERT COMONFOR.INS
C
INTEGER I,ISLOT,ISLOT2
C
C CALLING LOAD MODEL
C I = LOAD ARRAY INDEX
C ISLOT = SHARED DATA ARRAY ALLOCATION TABLE INDEX
C ISLOT2 = PRIVATE DATA ARRAY ALLOCATION TABLE INDEX
C J=LDSTRT(1,ISLOT) USE CON(J)
C K=LDSTR2(1,ISLOT) USE STATE(K) AND STATE(K+1)
C
INTRINSIC ABS, AIMAG, CONJG, REAL, CMPLX
C
INTEGER IB, J, K
REAL VM, T, U, A(25), B(10), C(10), PNEW, QNEW
COMPLEX PQNEW, PQOLD, PQOLD0, S
C
IF (MODE.GE.4) RETURN
RETURN
ENTRY TQUIBL (I,ISLOT,ISLOT2)
C
C BUS SEQUENCE NUMBER NEGATIVE IF LOAD IS OUT OF
SERVICE
C
IB = NUMLOD(I)
IF (IB.LE.0) RETURN
C
C GET STARTING 'CON' AND 'STATE'
C CON(J) - VOLT(0)
C CON(J+1) - P0
C CON(J+2) - Q0
C STATE - X1,X2
C
J = LDSTRT(1,ISLOT)
K = LDSTR2(1,ISLOT)
C
```

```
C INITIALIZE
C
C VM = ABS(VOLT(IB))
C
A = [0.98705, -0.011057, -0.011249, -0.0029144, 0.0057223,
&-0.0169, 0.98966, -0.0039157, -0.036558, -0.010731,
&-0.25402, -0.063949, 1.0466, 0.2293, -0.20354,
&0.087703, 0.002135, -0.18908, 0.30655, 0.18851,
&-0.55421, -0.09808, 0.22954, 0.80555, 0.46343]
B = [-0.94518, -82.864,
&-2.1867, -135.4,
&85.381, 8563.2,
&-137.44, -12818.0,
&245.19, 24109.0]
C = [-289.04, 167.28, -1.4629, -3.2845, -1.5786,
&-30.518, -232.24, 4.5873, 4.6307, 0.82434]
C
C CALCULATE DESIRED TOTAL LOAD INJECTION
C
IF (TIME.LE.0) THEN
CON(J) = ABS(VOLT(IB))
VM=ABS(VOLT(IB))
PQOLD0 = CLODFR(1,I)+VM*CLODFR(2,I)+VM*VM*CONJG(CLODFR(3,I))
CON(J+1) = REAL(PQOLD0)
CON(J+2) = AIMAG(PQOLD0)
CON(J+3) = BSFREQ(IB)
STATE(K) = 0
STATE(K+1) = 0
STATE(K+2) = 0
STATE(K+3) = 0
STATE(K+4) = 0
STATE(K+5) = 0
TPLOAD(I) = REAL(PQOLD0)
TQLOAD(I) = AIMAG(PQOLD0)
END IF
IF (TIME.GT.0) THEN
PNEW=0.01*(C(1)*STATE(K)+C(2)*STATE(K+1)+C(3)*STATE(K+2)
&+C(4)*STATE(K+3)+C(5)*STATE(K+4))+CON(J+1)
QNEW=0.01*(C(6)*STATE(K)+C(7)*STATE(K+1)+C(8)*STATE(K+2)
&+C(9)*STATE(K+3)+C(10)*STATE(K+4))+CON(J+2)
C
STATE(K) =A(1)*STATE(K)+A(2)*STATE(K+1)
&+A(3)*STATE(K+2)+A(4)*STATE(K+3)+A(5)*STATE(K+4)
&+B(1)*(ABS(VOLT(IB))-CON(J))+B(2)*(BSFREQ(IB)-CON(J+3))
STATE(K+1)=A(6)*STATE(K)+A(7)*STATE(K+1)
&+A(8)*STATE(K+2)+A(9)*STATE(K+3)+A(10)*STATE(K+4)
&+B(3)*(ABS(VOLT(IB))-CON(J))+B(4)*(BSFREQ(IB)-CON(J+3))
STATE(K+2)=A(11)*STATE(K)+A(12)*STATE(K+1)
&+A(13)*STATE(K+2)+A(14)*STATE(K+3)+A(15)*STATE(K+4)
```

```
&+B(5)*(ABS(VOLT(IB))-CON(J))+B(6)*(BSFREQ(IB)-CON(J+3))
STATE(K+3)=A(16)*STATE(K)+A(17)*STATE(K+1)
&+A(18)*STATE(K+2)+A(19)*STATE(K+3)+A(20)*STATE(K+4)
&+B(7)*(ABS(VOLT(IB))-CON(J))+B(8)*(BSFREQ(IB)-CON(J+3))
STATE(K+4)=A(21)*STATE(K)+A(22)*STATE(K+1)
&+A(23)*STATE(K+2)+A(24)*STATE(K+3)+A(25)*STATE(K+4)
&+B(9)*(ABS(VOLT(IB))-CON(J))+B(10)*(BSFREQ(IB)-CON(J+3))
STATE(K+5) = STATE(K+5)+1
C
PQNEW = CMPLX(PNEW,QNEW)
TPLOAD(I) = REAL(PQNEW)
TQLOAD(I) = AIMAG(PQNEW)
VM=ABS(VOLT(IB))
PQOLD = CLODFR(1,I)+VM*CLODFR(2,I)+VM*VM*CONJG(CLODFR(3,I))
CURNT(IB) = CURNT(IB)-CONJG((PQNEW-PQOLD)/VOLT(IB))
END IF
RETURN
END
```

Single Input Model

```
SUBROUTINE EQUIBL(I,ISLOT,ISLOT2)
C
$INSERT COMONFOR.INS
C
INTEGER I,ISLOT,ISLOT2
C
C CALLING LOAD MODEL
C I = LOAD ARRAY INDEX
C ISLOT = SHARED DATA ARRAY ALLOCATION TABLE INDEX
C ISLOT2 = PRIVATE DATA ARRAY ALLOCATION TABLE INDEX
C J=LDSTRT(1,ISLOT) USE CON(J)
C K=LDSTR2(1,ISLOT) USE STATE(K) AND STATE(K+1)
C
INTRINSIC ABS, AIMAG, CONJG, REAL, CMPLX
C
INTEGER IB, J, K
REAL VM, T, U, A(25), B(5), C(10), PNEW, QNEW
COMPLEX PQNEW, PQOLD, PQOLD0, S
C
IF (MODE.GE.4) RETURN
RETURN
ENTRY TQUIBL (I,ISLOT,ISLOT2)
C
C BUS SEQUENCE NUMBER NEGATIVE IF LOAD IS OUT OF
SERVICE
C
IB = NUMLOD(I)
```

```
IF (IB.LE.0) RETURN
C
C GET STARTING 'CON' AND 'STATE'
C CON(J) - VOLT(0)
C CON(J+1) - P0
C CON(J+2) - Q0
C STATE - X1,X2
C
J = LDSTRT(1,ISLOT)
K = LDSTR2(1,ISLOT)
C
C INITIALIZE
C
C VM = ABS(VOLT(IB))
C
A = [1.0028, 0.001144, 0.021375, -0.0016845, 0.00067725,
&0.00084842, 0.9716, 0.0036193, -0.030211, 0.0059913,
&-0.023264, 0.04294, 0.97511, 0.078294, 0.039979,
&0.031067, -0.16589, 0.10478, 0.72387, 0.087577,
&-0.0094203, 0.066478, -0.035102, 0.0015887, 0.69002]
B = [0.091528, 0.65745, -1.0776, 6.5062, -3.7518]
C = [676.3, -215.8, 6.2354, 3.9756, 0.89492,
&-105.48, 298.7, -1.1342, -5.6304, -0.43385]
C
C CALCULATE DESIRED TOTAL LOAD INJECTION
C
IF (TIME.LE.0) THEN
CON(J) = ABS(VOLT(IB))
VM=ABS(VOLT(IB))
PQOLD0 = CLODFR(1,I)+VM*CLODFR(2,I)+VM*VM*CONJG(CLODFR(3,I))
CON(J+1) = REAL(PQOLD0)
CON(J+2) = AIMAG(PQOLD0)
STATE(K) = 0
STATE(K+1) = 0
STATE(K+2) = 0
STATE(K+3) = 0
STATE(K+4) = 0
STATE(K+5) = 0
TPLOAD(I) = REAL(PQOLD0)
TQLOAD(I) = AIMAG(PQOLD0)
END IF
IF (TIME.GT.0) THEN
PNEW=0.01*(C(1)*STATE(K)+C(2)*STATE(K+1)+C(3)*STATE(K+2)
&+C(4)*STATE(K+3)+C(5)*STATE(K+4))+CON(J+1)
QNEW=0.01*(C(6)*STATE(K)+C(7)*STATE(K+1)+C(8)*STATE(K+2)
&+C(9)*STATE(K+3)+C(10)*STATE(K+4))+CON(J+2)
C
STATE(K) =A(1)*STATE(K)+A(2)*STATE(K+1)
&+A(3)*STATE(K+2)+A(4)*STATE(K+3)+A(5)*STATE(K+4)
```

```
&+B(1)*(ABS(VOLT(IB))-CON(J))
STATE(K+1)=A(6)*STATE(K)+A(7)*STATE(K+1)
&+A(8)*STATE(K+2)+A(9)*STATE(K+3)+A(10)*STATE(K+4)
&+B(2)*(ABS(VOLT(IB))-CON(J))
STATE(K+2)=A(11)*STATE(K)+A(12)*STATE(K+1)
&+A(13)*STATE(K+2)+A(14)*STATE(K+3)+A(15)*STATE(K+4)
&+B(3)*(ABS(VOLT(IB))-CON
STATE(K+3)=A(16)*STATE(K)+A(17)*STATE(K+1)
&+A(18)*STATE(K+2)+A(19)*STATE(K+3)+A(20)*STATE(K+4)
&+B(4)*(ABS(VOLT(IB))-CON(J))
STATE(K+4)=A(21)*STATE(K)+A(22)*STATE(K+1)
&+A(23)*STATE(K+2)+A(24)*STATE(K+3)+A(25)*STATE(K+4)
&+B(5)*(ABS(VOLT(IB))-CON(J))
STATE(K+5) = STATE(K+5)+1
C
PQNEW = CMPLX(PNEW,QNEW)
TPLOAD(I) = REAL(PQNEW)
TQLOAD(I) = AIMAG(PQNEW)
VM=ABS(VOLT(IB))
PQOLD = CLODFR(1,I)+VM*CLODFR(2,I)+VM*VM*CONJG(CLODFR(3,I))
CURNT(IB) = CURNT(IB)-CONJG((PQNEW-PQOLD)/VOLT(IB))
END IF
RETURN
END
```

E MSE of Response in Machine Rotor Angle

MSE over 9s after the fault are listed in Table 1.

The responses of different type of equivalent models (listed in Table 8.3) under faults occurred at different locations (listed in Table 7.1) were simulated. These equivalents were derived using Fault 1. The MSE over 9s after the fault in rotor angles of individual generators in the transmission network for each equivalent model was calculated and listed in Table 1.

Fault	Generator	1(PQ)	2(eqr4)	3(eqr5)	4(eqr6)	5(peqr4)	6(peqr5)	7(peqr6)
Fault 1	G1	7.5959	4.8829	3.5804	9.6818	4.7763	76.302	14800
Fault 1	G2	8.0427	5.1771	3.85	10.11	5.0631	79.859	15401
Fault 1	G3	8.3302	5.4676	4.05	10.463	5.3513	81.966	16037
Fault 1	G4	9.7838	7.019	5.52	13.791	6.7894	102.02	15441
Fault 2	G1	7.5513	4.2906	4.98	5.7594	4.3559	7.5839	5.6731
Fault 2	G2	7.1284	4.0736	4.73	5.4785	4.138	7.3415	5.3951
Fault 2	G3	7.1204	4.1023	4.76	5.5096	4.1643	7.4663	5.4267
Fault 2	G4	7.1393	4.1318	4.7002	5.4742	4.185	8.4304	5.3984
Fault 3	G1	1.0243	0.090085	0.149	0.17892	0.084163	0.34321	0.16535
Fault 3	G2	1.039	0.095522	0.155	0.18598	0.089209	0.34589	0.1723
Fault 3	G3	1.0446	0.095876	0.155	0.18685	0.089539	0.34843	0.17304
Fault 3	G4	1.1347	0.14483	0.19012	0.27728	0.12576	0.51604	0.26395
Fault 4	G1	60.267	19.863	23.7	28.06	20.585	128.31	27.467
Fault 4	G2	59.919	20.17	24	28.205	20.931	133.09	27.62
Fault 4	G3	59.248	20.343	24.2	28.148	21.106	137.24	27.567
Fault 4	G4	57.941	21.156	24	27.968	21.835	160.05	27.45
Fault 5	G1	6.8603	0.17672	0.386	0.66814	0.16073	3.6897	0.60885
Fault 5	G2	6.8764	0.182 07	0.385	0.662	0.16458	3.7663	0.60385
Fault 5	G3	6.9055	0.19055	0.39	0.66395	0.17198	3.8573	0.60598
Fault 5	G4	7.2281	0.3138	0.46054	0.92883	0.25326	5.5745	0.8745

Table 1: Equivalent Models

F The Performance of Various Type of Equivalent under Faults at Different Locations

The equivalent models used in this section are listed in Table 8.3. Model 1 is the constant power equivalent model. Model 2-6 are state-space dynamic equivalent models with different orders derived from data with single input V. Among them, Model 2-4 are derived using N4SID algorithm and Model 5-6 are derived using PEM algorithm.

The original system model and the equivalent-based system models have been tested under the faults listed in Table 7.1. Figure 6.2 shows on the map the rough locations of these faults and the locations of the generators (G1 to G4) we are looking at in the transmission system.

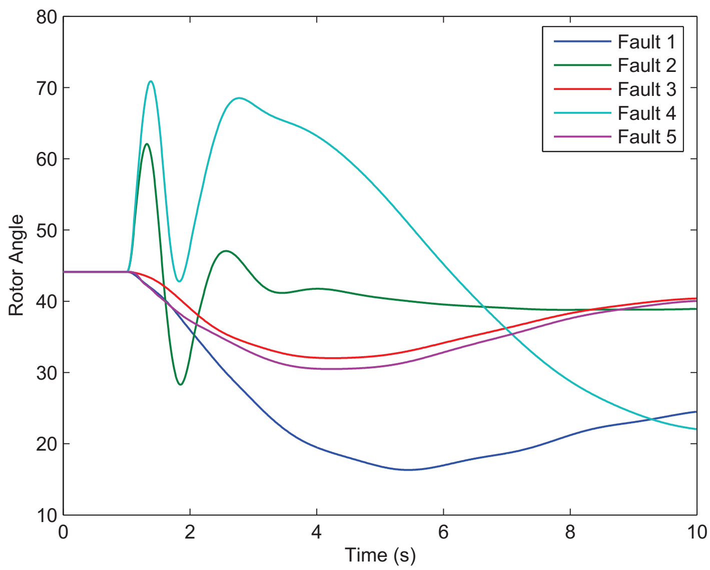
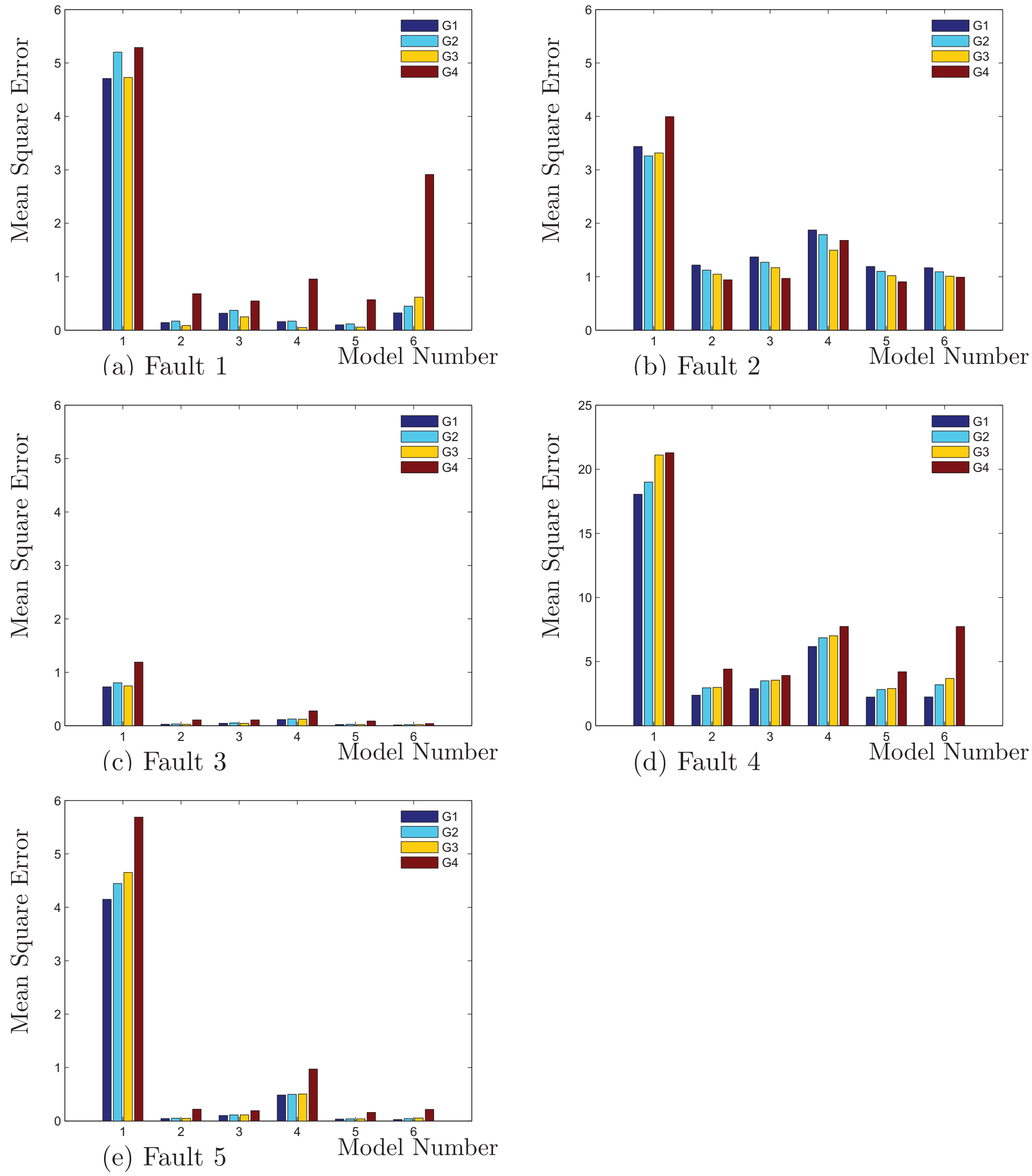


Figure 1: Rotor angle of machine G1 under various line faults

Mean Square Error

The responses of the original system in the rotor angle of machine $G1$ under different faults are shown in Figure 1.

The responses of all the equivalent models in rotor angle of machines in the transmission network under the same faults were obtained and their MSE to responses of the original system were calculated as introduced in section 7.4. These MSE values are the responses over 2s after each fault. The equivalent models include the constant power model (Model 1) and the dynamic equivalent models, which were derived from the system responses under Fault 1 (Model 2-6).



(a) Fault 1
(b) Fault 2
(c) Fault 3
(d) Fault 4
(e) Fault 5

Figure 2: Comparison of the equivalent models in rotor angle MSE of different machines (G1-G4) under different faults: Model 1, Constant power model; Model 2, 4^{th} order N4SID dynamic equivalent model; Model 3, 5^{th} order N4SID dynamic equivalent model; Model 4, 6^{th} order N4SID dynamic equivalent model; Model 5, 4^{th} order PEM dynamic equivalent model; Model 6, 5^{th} order PEM dynamic equivalent model.

Figure 2 shows the MSE of the machine angles for equivalent models in $2s$ response under different faults. The comparison of graph(a)-(e) indicates the association of the equivalent performances with the location of the fault. The results show the facts as follows:

- All the equivalent models perform worst in MSE under Fault 4 (shown in Figure 2 graph(d)), which causes the largest rotor angle variation in the system. The second largest rotor angle variation (under Fault 2) is corresponding to the second worst performances of the models (shown in Figure 2 graph(b)). This suggests that the larger the rotor angle variation caused by the fault, the worse the model performances in fitting the original model response in the first $2s$.

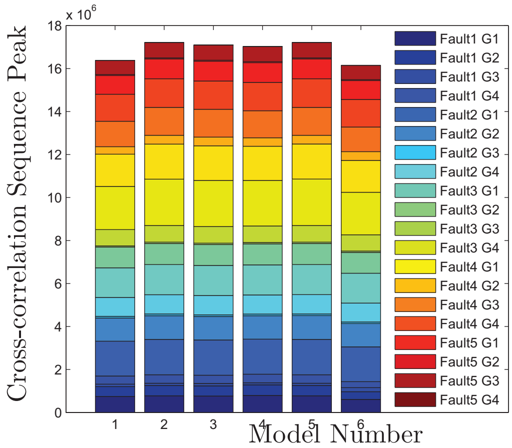
- The smaller the variation in rotor angle, the relatively better the state-space models are than the constant power model. For example, as can be seen from Figure 1, G1 has a small rotor angle variation under Fault 3 and a big one under Fault 4. Under Fault 3 the MSE of constant power model (Model 1) is 1.0243 and that of the 4th order N4SID model (Model 2) is 0.090085, about 11 times smaller. Under Fault 4 the former is 60.267 and the latter is 19.863, about 3 times smaller.

- When the fault locates in a long distance from the equivalent model (such as Fault 2), the MSE value of generator G4 (which locates very closely to the equivalent) and those of other generators under the same fault are relatively similar. When the fault happened close to the equivalent (such as Fault 1), the MSE value of G4 is more different from those of other machines. The reason for this might be that these equivalent models are linear models. If the electrical distance from the fault to the original distribution system model is significant, it's more reasonable to represent it as a linear model. When the fault is too close to the distribution system, the non-linear feature of the distribution system makes difference. To the generators in the transmission system, the further they are, the less significant non-linear effects the distribution system may have on them.

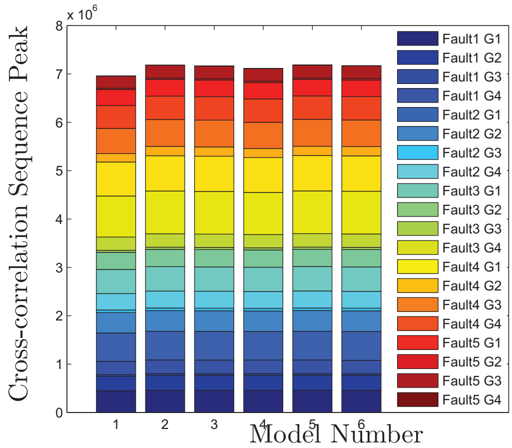
Cross-Correlation Sequence Peak of Rotor Angle

Figure 3 shows the sum of the cross-correlation sequence peak values of each equivalent model systems to the original distribution system model under 5 different faults on 4 different machines over 9s and 2s respectively. This figure suggests the general performances of the equivalent models have the following feature:

- For both the responses over two different time length, the equivalents have similar cross-correlation sequence peak values. The cross-correlation sequence peak value differences between the equivalent models are insignificant.



(a) Over 9s after the fault



(b) Over 2s after the faul

Figure 3: Comparison of the equivalent models in rotor angle cross-correlation sequence peak value of different machines (G1-G4) under different faults (Fault 1-Fault 5): Model 1, Constant power model; Model 2, 4^{th} order N4SID dynamic equivalent model; Model 3, 5^{th} order N4SID dynamic equivalent model; Model 4, 6^{th} order N4SID dynamic equivalent model; Model 5, 4^{th} order PEM dynamic equivalent model; Model 6, 5^{th} order PEM dynamic equivalent model.

- Most dynamic equivalent models (except Model 6 over 9s after the fault) have larger cross-correlation sequence peak values than that of a constant power model, which indicates that they have better performances than the latter.
- For both the responses over two different time length, the two 4th-order dynamic equivalent models have the best performances among the all.
- For models with the order higher than 4th, the N4SID state-space models behave better than the PEM state-space model.
- For each type of dynamic equivalent model, with the orders range from 4th to 6th, the lower order the model has, the better its performance is .

Figure 4 shows the cross-correlation sequence peak value of the machine angles responses of the equivalent models in $2s$ after each of the 5 faults. This figure helps to line out the association of the performances of the equivalent models in cross-correlation sequence peak value and the locations of the fault. From this figure it can be seen that:

- Regardless where the fault locates and what equivalent model is used, the cross-correlation sequence peak value always satisfies the inequation below:

$$G1 > G2 > G4 > G3$$

  This order is also the order of these generators in their absolute rotor angles. In Mean Square Error (MSE), however, the performances of

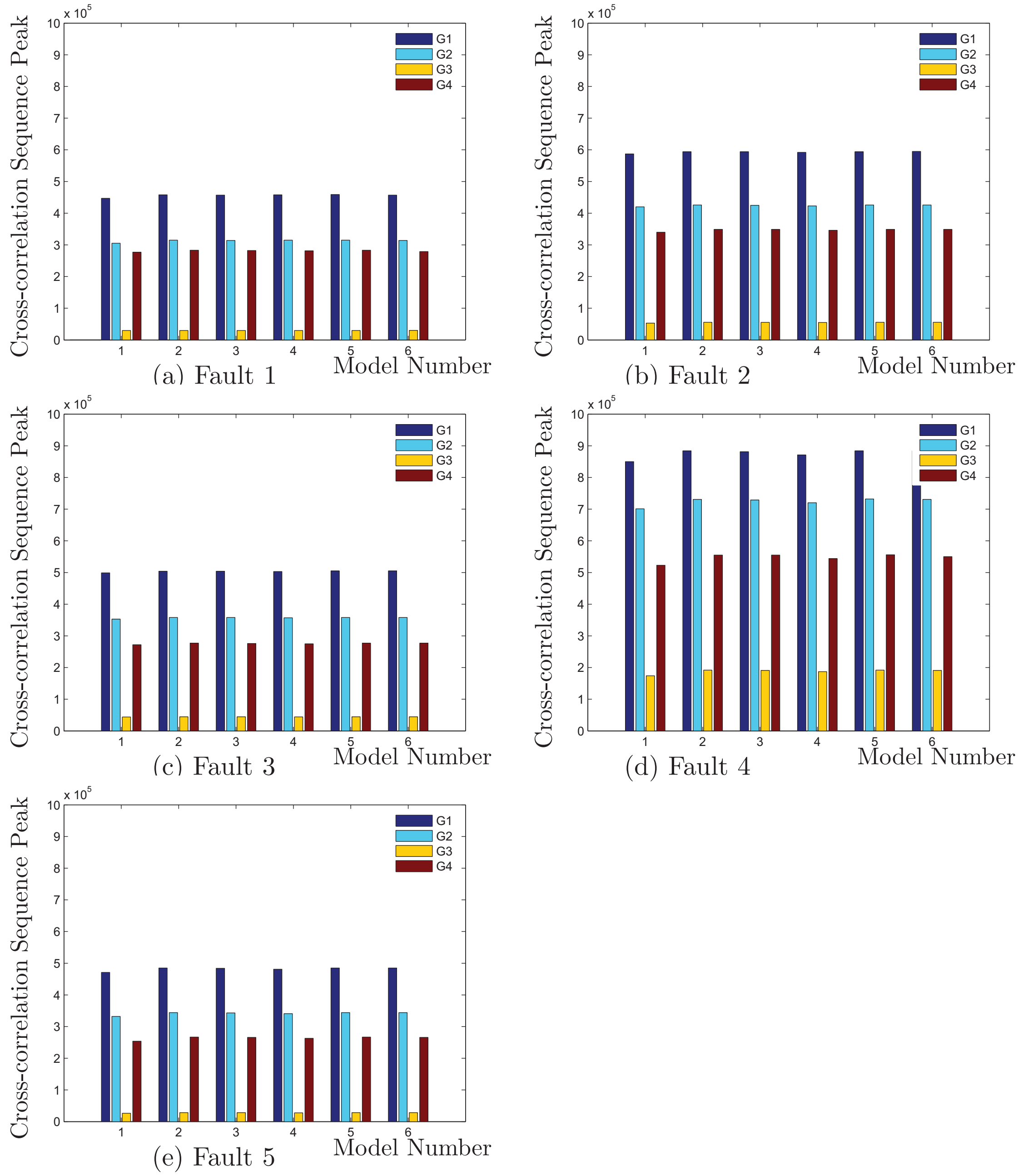


Figure 4: Performance of dynamic equivalent models derived by Fault 1 in rotor angles of different machines (G1-G4) under different faults: Model 1, Constant power model; Model 2, 4^{th} order N4SID dynamic equivalent model; Model 3, 5^{th} order N4SID dynamic equivalent model; Model 4, 6^{th} order N4SID dynamic equivalent model; Model 5, 4^{th} order PEM dynamic equivalent model; Model 6, 5^{th} order PEM dynamic equivalent model.

the equivalent models are not in a fixed order. Machine $G3$ always has the worst performance in cross-correlation sequence peak value but not always the worst in MSE. The fact of this suggests that the responses of the equivalents on machine $G3$ have relatively small phase shift than those on machine $G1$, $G2$ and $G4$.

- The diversity of the equivalent model performances in cross-correlation sequence peak value is small. Whilst the diversity of the equivalent model performances in MSE is relatively big. This shows that if there is no such phase shifts, the constant power model should have more similar performances to the dynamic equivalents. The difference between the constant power model and the dynamic equivalent models in MSE is partly caused by phase shift. This indicates that the dynamic equivalents may have the contribution in adjusting the phase shift, which was caused by eliminating the distribution network from the whole network.
- In Figure 2, the results show that in MSE, the response of equivalent models on machine $G4$ is always the worst except that under fault 2. This suggests that the equivalent model responses on machine G4 may have obvious phase shifts after the distribution network was replaced. G4 is a machine locates quite close to the distribution network. Hence it can be affected more than other machines by this replacement. Fault 2 locates quite far from the distribution network. Under Fault 2, replacing the distribution network with the equivalents does not cause more obvious phase shift on $G4$ than on other machines. The possible reason for this can be that the response of the distribution network is small under a faraway fault. Also, a linear model have more reasonable basis in this case.

Summary

The above discussions are on the performances of the equivalence models under the fault that was used to derived the equivalent model. By analysing the performances of the equivalent models, which was derived with the responses to a same fault, under different faults, we can find out that, no matter where the fault locates, the dynamic equivalent models always perform better than the constant power model and it may adjust the phase shift that caused by eliminating the distribution network.

G Embedded Conventional Generation

G.1 Machine G2 Response under Fault 2

The modes identified from the rotor angle response of $G2$ under Fault 2 (see Figure 5) are shown in figure 6.

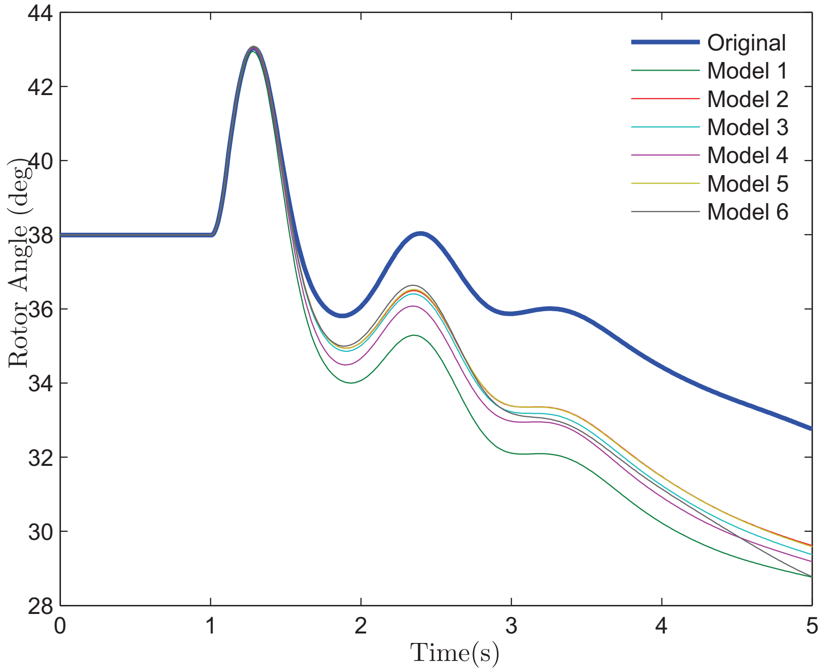


Figure 5: Dynamic responses in rotor angle of $G2$ under Fault 2

In Figure 6, the dynamic equivalent models are better at representing the interarea mode positions than the constant power model. Fault 8 is far from the distribution, which makes it more reasonable to use the linear model in application.

G.2 Machine G3 Response under Fault 2

The modes identified from the rotor angle response of $G3$ under Fault 2 (see Figure 8)are shown in figure 9. $G3$ is neither close to Fault 1 or Fault 2 nor to the distribution system. Hence the mode positions of the constant power and most dynamic equivalent models are quite similar to the original system.

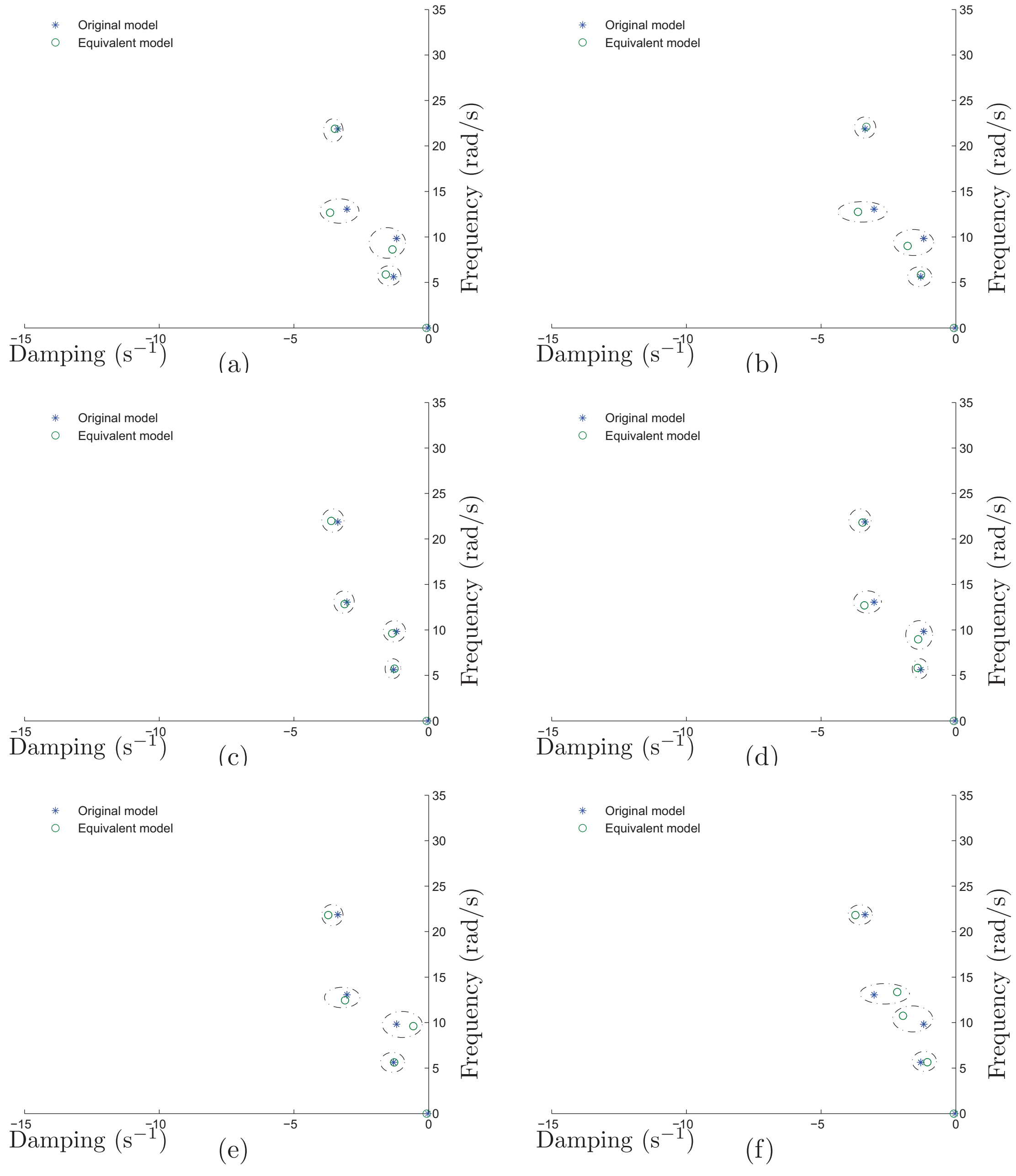
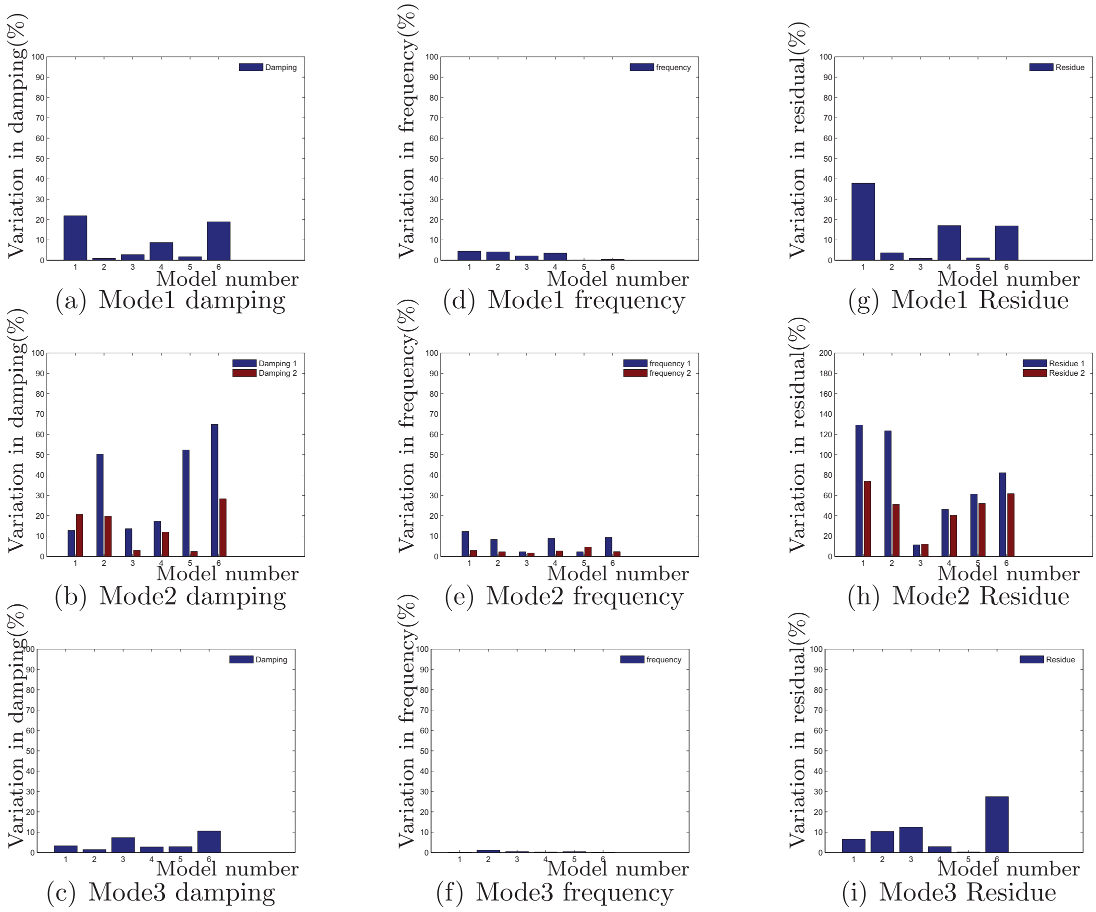


Figure 6: Rotor angle of Machine G2 under Fault 2 when using the equivalent models derived using Fault 1 - Modes identified by Prony analysis: (a) Constant power; (b) 4th order N4SID model; (c) 5th order N4SID model; (d) 6th order N4SID model; (e) 4th order PEM model; (f) 5th order PEM model



(a) Mode1 damping (d) Mode1 frequency (g) Mode1 Residue

(b) Mode2 damping (e) Mode2 frequency (h) Mode2 Residue

(c) Mode3 damping (f) Mode3 frequency (i) Mode3 Residue

Figure 7: Rotor angle of Machine G2 under Fault 2 when using the equivalent models derived using Fault 1 - Variation in damping, frequency and residue of modes identified by Prony analysis: Model 1, Constant power model; Model 2, 4^{th} order N4SID dynamic equivalent model; Model 3, 5^{th} order N4SID dynamic equivalent model; Model 4, 6^{th} order N4SID dynamic equivalent model; Model 5, 4^{th} order PEM dynamic equivalent model; Model 6, 5^{th} order PEM dynamic equivalent model.

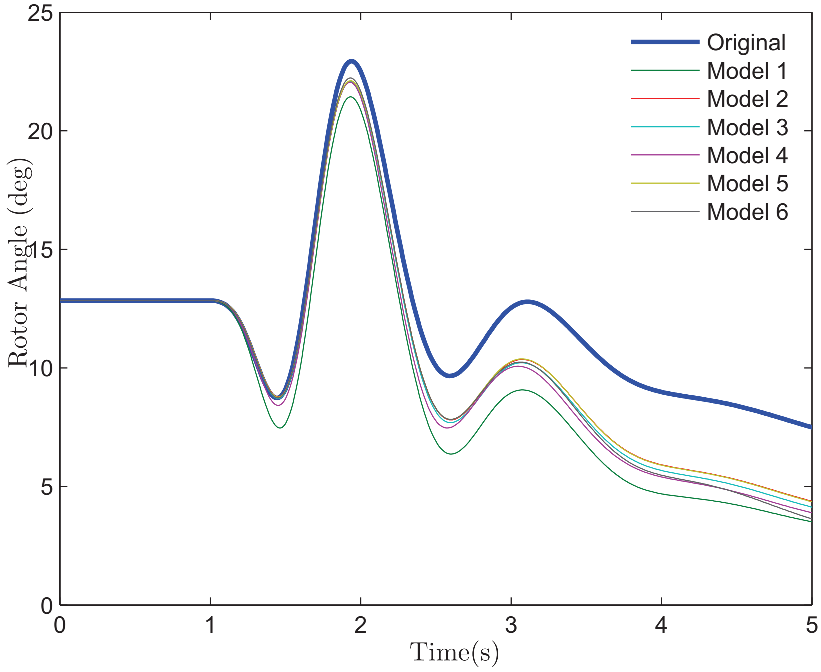


Figure 8: Dynamic responses in rotor angle of $G3$ under Fault 2

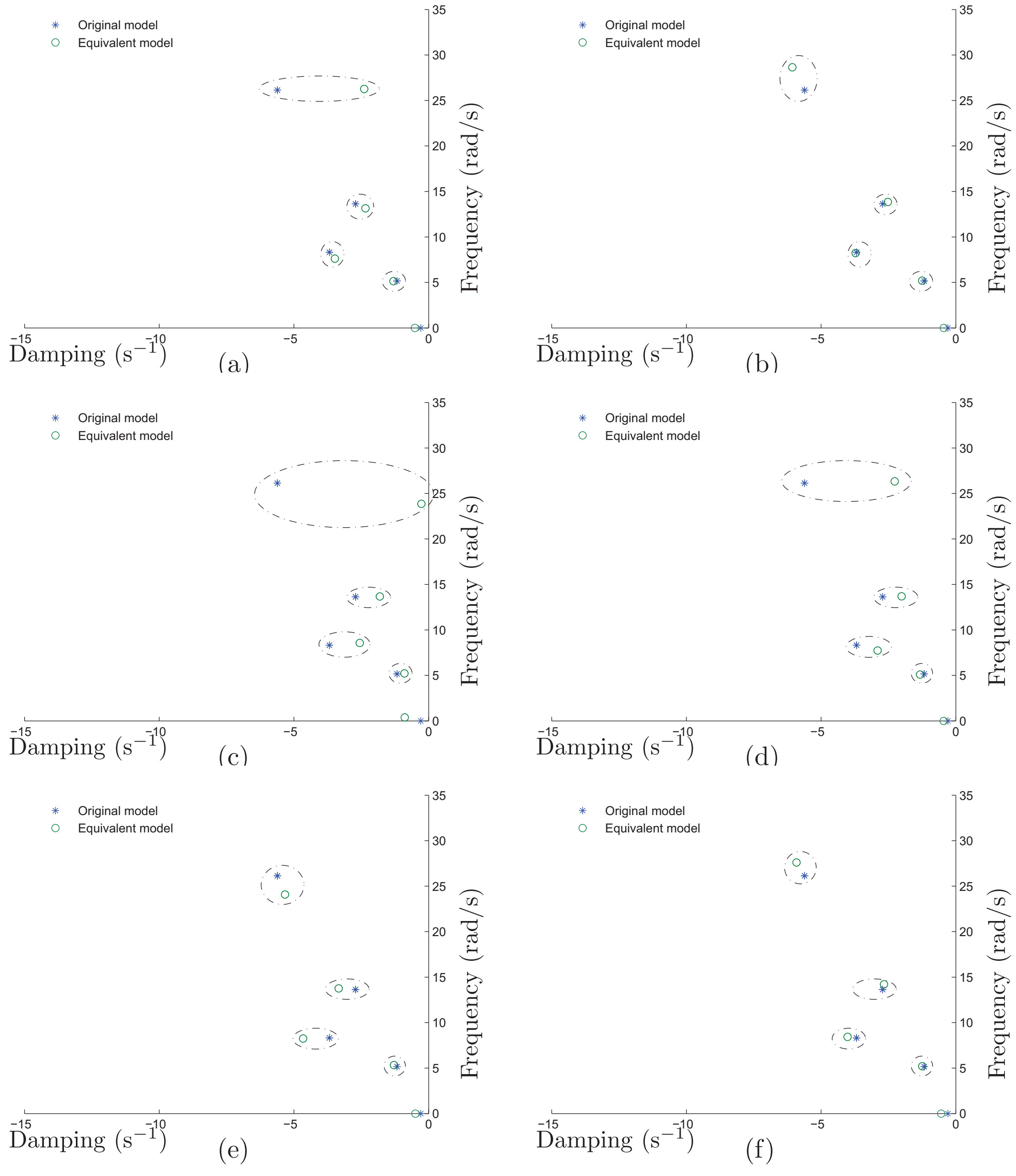
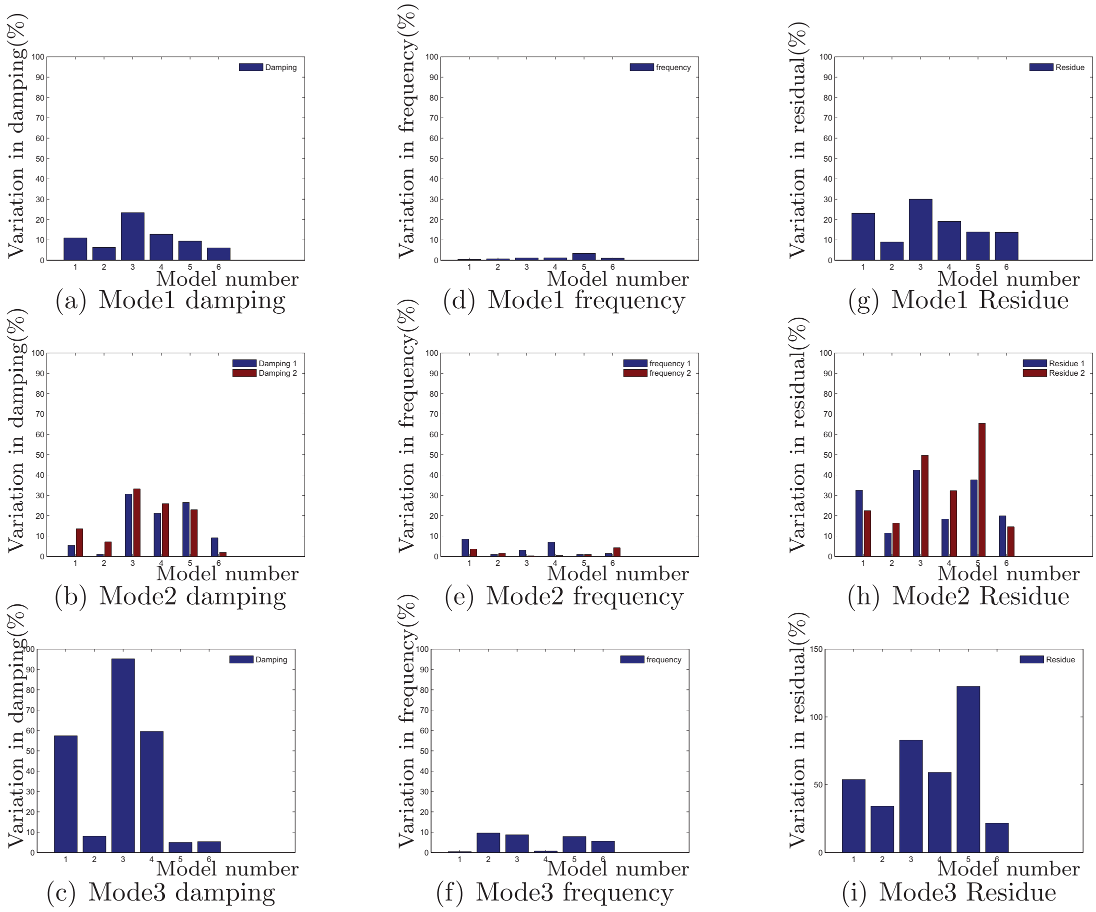


Figure 9: Rotor angle of Machine G3 under Fault 2 when using the equivalent models derived using Fault 1 - Modes identified by Prony analysis: (a) Constant power; (b) 4th order N4SID model; (c) 5th order N4SID model; (d) 6th order N4SID model; (e) 4th order PEM model; (f) 5th order PEM model



(a) Mode1 damping (d) Mode1 frequency (g) Mode1 Residue

(b) Mode2 damping (e) Mode2 frequency (h) Mode2 Residue

(c) Mode3 damping (f) Mode3 frequency (i) Mode3 Residue

Figure 10: Rotor angle of Machine G3 under Fault 2 when using the equivalent models derived using Fault 1 - Variation in damping, frequency and residue of modes identified by Prony analysis: Model 1, Constant power model; Model 2, 4^{th} order N4SID dynamic equivalent model; Model 3, 5^{th} order N4SID dynamic equivalent model; Model 4, 6^{th} order N4SID dynamic equivalent model; Model 5, 4^{th} order PEM dynamic equivalent model; Model 6, 5^{th} order PEM dynamic equivalent model.

H Publications

DYNAMIC EQUIVALENCING OF DISTRIBUTION NETWORK WITH HIGH PENETRATION OF DISTRIBUTED GENERATION

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ABSTRACT

Bulk connection of renewable energy sources to the existing distribution networks in the recent years has resulted in high increase of the power systems complexity. This also has changed (and will be changing) dynamic properties of the power systems. Both of these factors lead to the increasing size of the power system models, which are essential to analysis of the power system operation and stability problems. Due to the difficulties related to obtaining data for these models, dynamic equivalencing becomes an interesting solution to the future power systems modelling. Proper equivalencing method is supposed to maintain the main dynamic characteristics of a power system whilst optimally (reasonably) reducing its size. This paper focuses on deriving dynamic equivalents of distribution network taking distributed generators into consideration. The dynamic equivalents are established by system identification method.

Keywords: System Identification, Dynamic Equivalent, Distributed Generation

1 INTRODUCTION

The power system models are the basic tools for various types of the power system operation analysis, e.g. stability analysis, designing of control systems, verification of designed controllers, etc. Example of the power system model usage for analysis purpose (the life of a power system model) is presented in Figure. 1. In general and in theory the full model of the power system can be used in the all pointed stages of the model utilisation. However, in practice, building, maintaining, and updating of such a model is an extremely difficult task. Due to the increasing size and complexity of the nowadays power systems, it is almost impossible to study the dynamics and stability problems of a full power system model. A solution for this is equivalent. Figure 1 also shows the place that a dynamic equivalent of the power system model could be used.

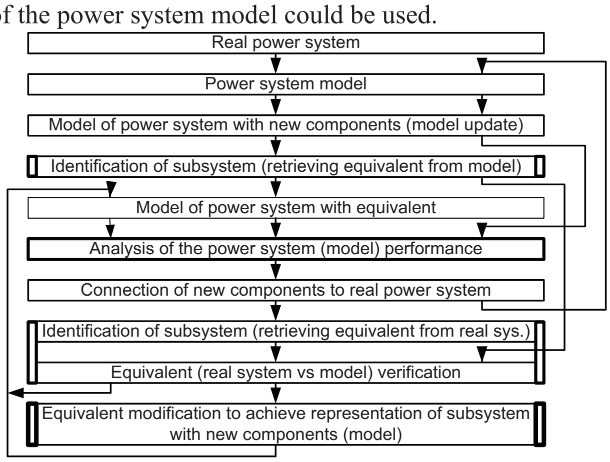

Figure 1 Power system model and equivalent usage for research and analysis

In recent years, various methods for power systems static and dynamic equivalencing have been developed. Dynamic equivalencing is to eliminate a part of the network and replace it by an equivalent model (Figure. 2), which has enough close dynamic characteristics to the original (full) model. Node elimination method [1] relies on modelling loads by constant impedances and eliminating them by using a Ward equivalencing technique. Modal analysis method [2] relies on simplification of the system by aggregating similar modes and by eliminating modes not common to the similar mode group. Coherency-based method [3] relies on identification of coherent generators (generators that tend to swing together after a disturbance) and aggregation them into a single equivalent generator.

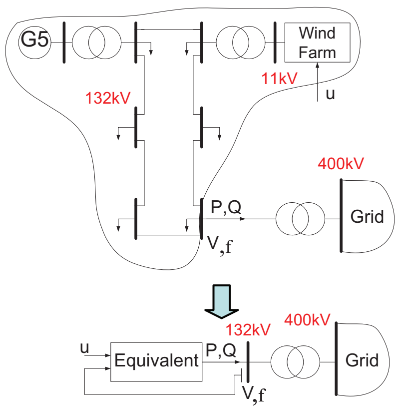

Figure 2 Idea of a subsystem equivalencing

For all the methods mentioned above, detailed information of network structure and parameters is required. However, in a real power system, such information would not be accessible, especially in case of wind turbines and other small generating units.

Identification method proposed in this paper could solve this problem by regarding the system as a black box. It directly derives equivalent models of the power system based on some observed Input/Output data. Therefore, the detailed information of the system that needs to be reduced is not necessarily required. This method can be applied to real power systems, which means that the measurements from real system could be used to create equivalent for a given subsystem.

2 IDENTIFICATION OF DYNAMIC SYSTEM

The proposed method allows to identify parameters of a selected model structure based only on measured data from a chosen point of the system. PSS/E program is initially used to simulate the dynamic behaviour of the power system. Different types of disturbances are introduced to the power system model. Measurements are made after disturbances at the connecting point of distribution network model and transmission network model (Figure. 2). Time series of voltage and frequency are used as inputs and time series of real power and reactive power are used as outputs. These data is imported to Matlab System Identification toolbox for the parameter identification. Mathematical model structures like ARX, State-Space, etc. are used in this toolbox.

2.1 Inputs to the model (equivalent)

In this project, voltage and frequency at the connecting point of transmission network and distribution network have both been used as input signals.

The static and dynamic characteristics of the system have to be considered. The static frequency characteristics of loads like cities or areas are close to linear while voltage characteristics are closer defined by square function. Small synchronous generators and wind generators produce power that does not depend on frequency. Their voltage characteristics are then linear. Bigger generating units usually have frequency dependent and voltage independent (voltage control) characteristics.

Dynamic characteristics of the system are complex. The loads and generating units response depends both on frequency and voltage. Synchronous machines are low damped objects while asynchronous machines are high damped. Then in general the synchronous machines can mainly influence the dynamic properties of the power system. But of course, all depends here on structure of a given power system.

For a complex multi-objects system, with unknown parameters, the dynamic properties of the system can only be obtained from measurements. In this project, studies focus on the equivelencing with unknown structure. So the measurements of frequency and voltage have been used as inputs.

In practice, to obtain data using measurement, problems are increased by measuring frequency because of the noise, especially when the disturbance applied to the system is relatively small. Therefore the measured signal variation in magnitude could be close to the noise. For this reason, single input (voltage) has also been tested to derive equivalents. However, the results confirm that the state of the system (distribution network) depends on both of these factors.

Wind velocity would also be a useful input for the models with wind turbines, especially for the interests in power system responses to wind changes.

2.2 Disturbances

The disturbances could be grouped into three: disturbances in transmission network; disturbances in distribution network; and disturbances at the connecting bus. The object system that we want to derive equivalent of is the distribution network. In reality we cannot disconnect the distribution network from the transmission network for testing and measuring. Therefore the data sets measured at the connecting bus actually include the responses of not only the distribution network but also the transmission network. The key point is to find out the disturbances, which have minor effects on transmission network so that the dynamic response of the system is mainly the response of the distribution network. For this reason, if we choose disturbances in transmission network, the generator oscillations in transmission network cannot be neglected. So the models that we derive using these measurements are the equivalents of the full system (transmission network and distribution network) rather than distribution network only. Some disturbances in distribution network may have small effects on transmission network. Faults like short-circuit, tripping branch and closing branch etc. have been simulated in distribution network using PSS/E for collecting data. The disadvantage of adding disturbances in distribution network is that it could change the topology of the object system during the fault time. This will more or less affect the accuracy of equivalents. Also, these equivalents are highly dependent on the location of the disturbance. The third way is to add disturbances at the bus, which connects transmission network and distribution network. Among the disturbances that could be added at the connecting bus, changing loads may have relatively small effects on transmission network.

To disconnect load and reconnect load could keep the after-fault steady state the same as the initial steady state. On the other hand, the sole load disconnection would change the steady state value. In this case, the dynamic response of the network is the combination of the response to the load change and a step response. However, from the practice point of view, load disconnection and reconnection could be tested in the real power system without affecting the network operation very much, if the disconnection time is short enough.

The test system used for this project is shown in Figure 3 with 6 generating units listed in Table 1.

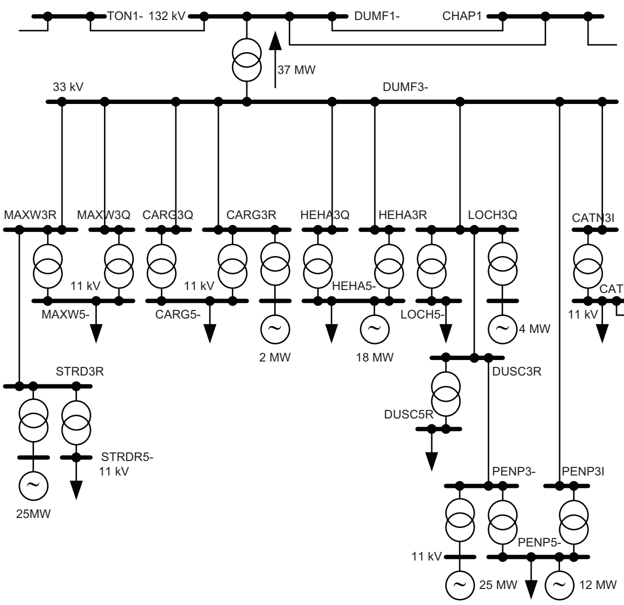

Figure 3 Distribution power system connected to Dumfries 132 kV (DUMF1-) substation.

Table 1: Give title

Bus	Unit type	S_n [MVA]	P_g [MW]
STRD3R	Steam	30	25
PENP3-	Steam	30	25
CARG3R	Diesel	2.5	2
LOCH3Q	Diesel	5	4
HEHA5-	Hydro	20	18
PENP5-	Hydro	14	12

Figure 4 shows the responses of 2 generating units in transmission network and 2 generating units in distribution network after disconnecting a reactive load (2 Mvar). Please note the unit of real power in transmission network is 10^{-5} p.u. That means the oscillations of the generators are really small, just as some noise. However, in the distribution network the generators have relatively obvious oscillations (the unit is 10^{-3} p.u. instead). In this case, we could suppose that using such disturbance, which has really small effects on transmission network, we could derive the dynamic equivalent, which represents only (mainly) the distribution network.

The load that was disconnected in the example is a reactive load. Real load and complex load disconnection have also been tested as disturbances. Disconnecting reactive loads is more practical in the real power systems since it could be done by just disconnecting bank capacitor. However, the real component of loads may have some effects on frequency. If the frequency has relatively small effects on the accuracy of the equivalent, the effects of real power on frequency could be neglected.

3 EQUIVALENCING OF DISTRIBUTION NETWORK

Identification of dynamic object is not an ideal and easy method, especially while applied to the nonlinear system, which the power system is. Various disturbances applied to the considered power system, various operating points will usually lead to various equivalents with best fitting to the measured time series.

There are dozens of model structures of the equivalent in Matlab identification toolbox, which can result in different model quality.

ARX model relates the current output $y(t)$ to a finite number of the past outputs $y(t-k)$ and inputs $u(t-k)$.

$$y(t) + a_1 y(t-1) + \dots + a_{na} y(t-na) =$$
$$b_1 u(t-nk) + \dots + b_{nb} u(t-nk-nb+1) \quad (1)$$

State-space model relates the current output $y(t)$ and the next state $x(t+1)$ to current state variables $x(t)$ and input $u(t)$. $e(t)$ is a white noise.

$$x(t+1) = Ax(t) + Bu(t) + Ke(t) \quad (2)$$
$$y(t) = Cx(t) + Du(t) + e(t) \quad (3)$$

These two models have been used in this project. They perform differently when disturbances are different. It's hard (there are no theoretical ways) to say which model is the best in general. Therefore a wide choice of models with different orders has been tested for each disturbance to find out acceptably accurate equivalents.

4 VERIFICATION OF THE METHOD

Equivalents derived using system identification toolbox has been implemented into PSS/E user-defined models. Load-related model is chosen because it not only does the calculation of state variables but also allows the network solution calculate current injections which are dependent on the bus voltage.

The whole system was modelled in PSS/E program. Disturbances were introduced into either distribution network or connecting bus. Time series of real power, reactive power, voltage and frequency were measured at the connecting bus. These time series were then imported into Matlab System Identification Toolbox to derive equivalent models, which were later introduced into PSS/E to replace the distribution network for verification purpose.

Figure 5 shows the responses of an equivalent to load disconnection (a) and load disconnection-reconnection (b). In these power graphs, the solid blue lines (T+D

=Transmission network + Distribution network) are the full model response in PSS/E, the dotted red lines (Eq=Equivalent) are the identified fitting equivalents to original response curves in Matlab, and the dashed green lines (T+Eq=Transmission network + Equivalent of Distribution network) are the response of equivalents introduced into PSS/E program.

From the results shown in Figure 4, we can see that the oscillations of the generators last longer when load was only disconnected (a). This is because the configuration of the full network changed after the disturbance. So the steady-state also changed and introduced a step response into the system. For this reason, the equivalent derived using the measurement under this disturbance would be less accurate. Figure 5 (a) shows the obvious differences in reactive power response and in the voltage at steady state of the full system and the system with equivalent.

Using load disconnection and reconnection, we could derive relatively good equivalents. It can be seen in Figures 5 (b) that the responses of the full model and the equivalents are close to each other in voltage and their power oscillations are almost in phase. There is some difference in amplitude and the reasons for the existence of this difference could be: a) the accuracy of equivalent that we derived using identification method is not high enough in this example. This could be improved by finding out other better fitting models. b) The oscillations in the inputs caused by disturbances might be too small, which caused the difficulty to derive an accurate equivalent from the measurements. This could be improved by choosing different disturbances.

The limitation of this equivalencing method is that the equivalent is highly dependent on the types and locations of the disturbances. If the disturbance used for verification is quite different from that used to derive the equivalent, the accuracy of the equivalent will then become a little disappointing. The discussion on this dependence will be included in future work.

5 CONCLUSIONS

The paper presents initial results of novel equivalencing method for distribution network with unknown structure. It is quite useful especially when the network is complex (with renewable generations etc.). The method on extracting dynamic response of distribution network from the full system has been discussed and fully tested in PSS/E.

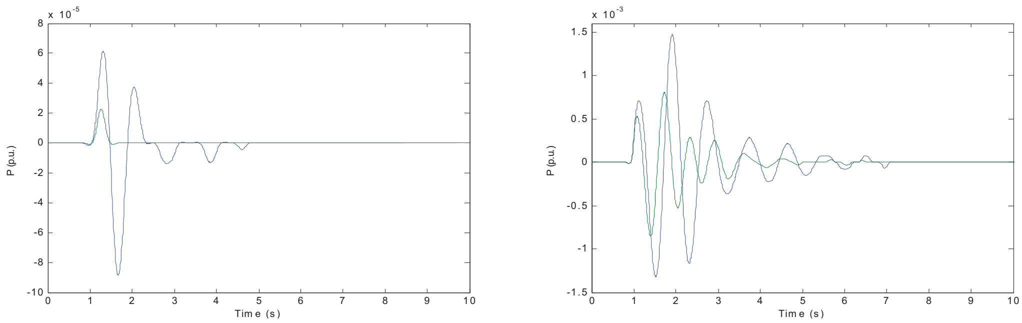

Figure 4(a) Units response in Transmission (left) and Distribution network (right) after load disconnection

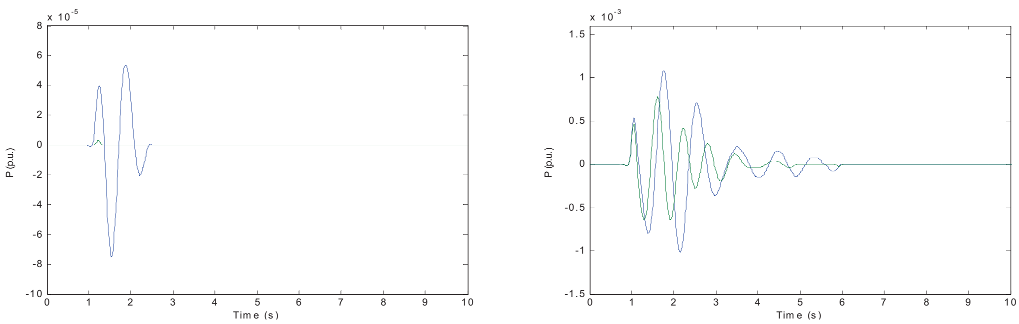

Figure 4(b) Units response in Transmission (left) and Distribution network (right) after load disconnection and reconnection

6 ACKNOWLEDGEMENTS

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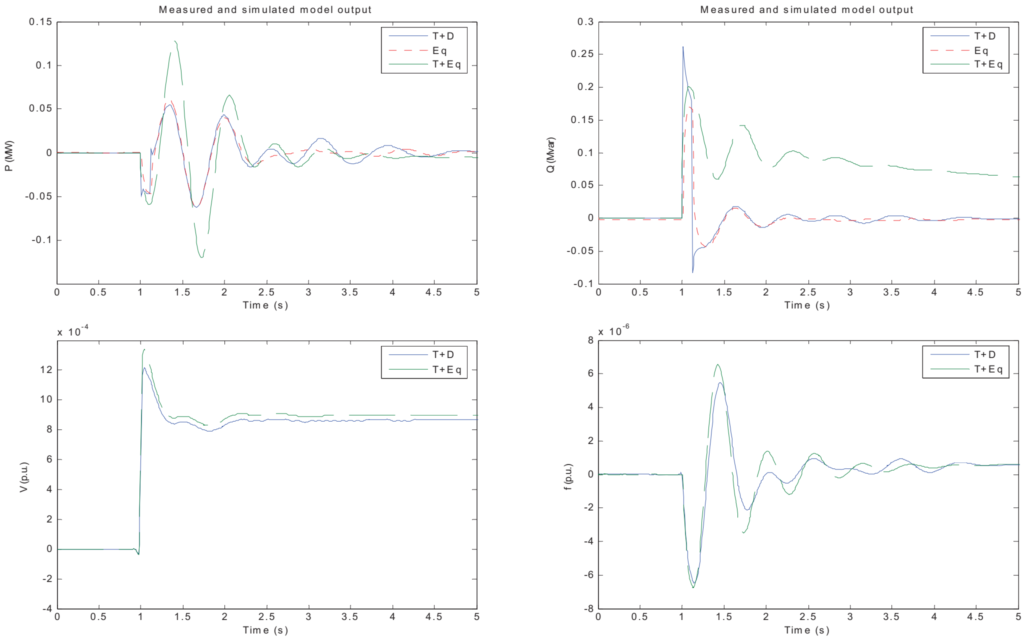

Figure 5(a) Response after load disconnection

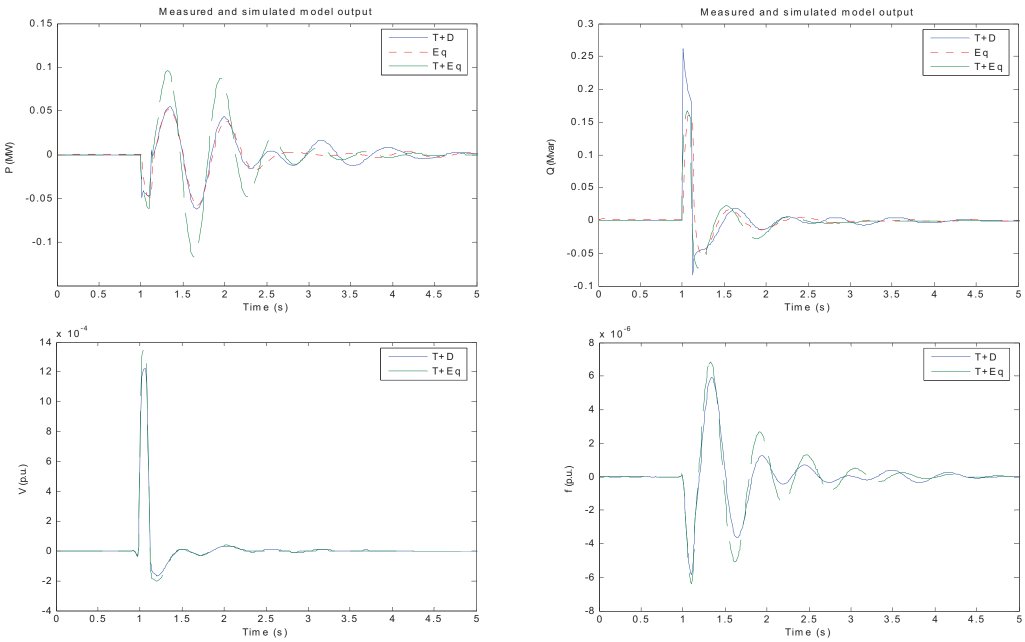

Figure 5(b) Response after load disconnection and reconnection

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Identification based Dynamic Equivalencing

X. Feng, Z. Lubośny, and J. W. Bialek

***Abstract*—Expected bulk connection of renewable energy sources to existing distribution networks may change dynamic properties of power systems and increase the size of power system models used for the analysis of power system operation and stability problems. To reduce the size and complexity of modelling, distribution networks could be replaced by their dynamic equivalents. This paper focuses on identification of dynamic equivalents of distribution network from measured disturbance data. The validity of derived state-space models has been confirmed by simulation.**

***Index Terms*— Dynamic Equivalent, System Identification, Distribution Network**

I. Introduction

Power system models are used for various types of power system operation analysis, e.g. stability analysis, designing of control systems, verification of designed controllers, etc. In practice, building, maintaining, and updating of such a model is an extremely difficult task. Traditionally, distribution networks tend to be replaced by their static equivalents in power system models used for dynamic system studies. However increased penetration of distribution networks by small generators, usually renewable ones, means that static equivalents cannot any longer represent correctly distribution networks. On the other hand, due to the increasing size and complexity of the whole transmission and distribution system, it is almost impossible to study the dynamics and stability problems using a full power system model. A solution for this is dynamic equivalencing.

Various methods for power systems static and dynamic equivalencing have been developed in the past. Node elimination method [1] relies on modelling loads by constant impedances and eliminating them by using the Ward equivalencing technique. Modal analysis method [2] relies on simplification of the system by eliminating modes, which have insignificant effects on system's dynamic behaviour. Coherency-based method [3] relies on identification of coherent generators (generators that tend to swing together after a disturbance) and aggregation them into a single equivalent generator.

The methods mentioned above are model-based, i.e. a detailed information of network structure, its parameters and models of individual generators is required. However, in a real distribution network, such information may not be accessible. The measurement-based identification method proposed in this paper could solve this problem by regarding the system as a black box. It directly derives an equivalent model of a power system based on some observed Input/Output data from disturbances. Therefore, the detailed information of the system model (i.e. its configuration, parameters, machine models etc.) that needs to be reduced is not necessarily required.

The identification method proposed in this paper is the offline state-space identification. The whole system was modelled in PSS/E program. A disturbance at a bus connecting the distributed network to be equivalenced with the transmission network was simulated and time series of real power, reactive power, voltage and frequency were measured and imported into Matlab System Identification Toolbox to derive an equivalent state-space model. The equivalent was then inserted into PSS/E using user-defined models to replace the distribution network for verification purpose. The results confirmed good performance of the methodology.

II. The Dynamic Equivalencing Problem

A. Definitions

For the purposes of this research, a complex interconnected power system can be divided into the study system and the external system. The study system is the part of the system which is of direct interests and where all the disturbances and configuration changes are assumed to happen. It should be retained in detail.

The external system is the rest of the system. It could be simplified to a reduced equivalent since most of the external systems have insignificant effects on the study system. And what we are interested in the external system are its effects on the study system.

The task of dynamic equivalencing is to eliminate the full model of the external system and replace it by an equivalent model, which has dynamic characteristics close enough to the full model.

B. Problems

1) Data Availability

For classical dynamic equivalencing methods, which are based on coherency or modal analysis, detailed information

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about network structure, parameters, and dynamic models of individual elements (generators) is required from the external system, which needs to be reduced. However, in reality such data of power systems for dynamic equivalencing is not always accessible. The data of the local network and generators may not be possible to obtain in detail (e.g. ragrading wind turbines and other small generating units). And the global data availability is also limited because other local operators would not want to reveal the specific data (e.g. the capability of power plants or the performance of loads) of their own network. For a complex multi-objects system with unknown parameters, the dynamic properties of the system can only be obtained from measurements.

2) Separation of system responses

In system identification methods, the parameters of equivalent model are derived by fitting the response of the system. But the external system and the study system are interconnected with each other so ideally the response of the external system (to be equivalenced) should be isolated from the response of the study system. It can easily done in simulation but such separation can not be done in practice. Hence, the system response, which is obtained after a disturbance, is the response of the <u>whole</u> system (a combined response of the external system and the study system). The equivalent derived using such a response, for this reason, is the equivalent of the full system instead of the equivalent of the external system only. Hence the main problem is how to choose a disturbance and the model so that the obtained equivalent models the external system only.

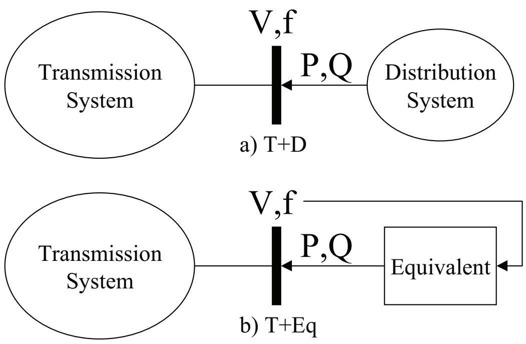

Fig.1. Description of dynamic equivalencing using system identification method.

C. Methodology

The dynamic equivalencing method presented in this paper is based on system identification, which identifies parameters of a selected model structure based on measurements from a chosen point of the system only. It contains three steps:

1) Data Collection

PSS/E program is initially used to simulate dynamic responses of the power system after various disturbances. Measurements are taken at the connecting bus between the external system model and the study system model (see Fig.1a). Time series of voltage and frequency are chosen as inputs and those of real power and reactive power are used as outputs.

2) Offline Identification

In this step, the collected input and output data are imported to Matlab for parameter identifications. The equivalents are derived in a form of mathematical model structures (e.g. discrete state-space model) using System Identification toolbox. The subspace algorithm used to derive the state-space models shown in this paper is described in Appendix.

3) Model Verification

The derived dynamic equivalent model, which has good performance in fitting the measurements, is then implemented as PSS/E user-defined models to replace the external system in the original power system model - see Fig. 1b. Verification is done by comparing responses to system disturbances (short-circuits) of the full model (Fig. 1a) and the reduced model (Fig.1b).

III. IDENTIFICATION OF DYNAMIC SYSTEM

A. Model Structure

Identification of a dynamic system is not an ideal and easy method, especially for a non-linear system such as the power system. Different disturbances applied to the considered power system may possibly lead to different equivalents with good fitting to the measured time series.

The selection of model structure results in different model performances. It is very difficult to say which model is the best in general. Therefore a wide choice of model structures with different orders has been tested to search for acceptably accurate equivalents.

B. Inputs to the Model

For a power system, the static and the dynamic characteristics should both be considered.

1) Static characteristics

a) Loads have static frequency characteristics close to linear and voltage characteristics closer defined by square function; b) Small synchronous generators and wind generators produce power, which does not depend on frequency. Their voltage characteristics are linear; c) Bigger generating units usually have frequency dependent and voltage independent (due to voltage control) characteristics.

2) Dynamic characteristics

Dynamic characteristics of the system are complex. a) The responses of loads and generating units depend on both frequency and voltage; b) Synchronous machines are poorly damped objects whilst asynchronous machines are better damped. In general dynamic properties of the power system are mainly influenced by synchronous machines and the structure of a given power system.

In this project, the measurements of voltage and frequency at the connecting bus between transmission network and distribution network are used as input signals. However, there are problems in measuring frequency due to the noise. Especially when the disturbance applied to the system is relatively small, the measured signal variation in magnitude could be close to the noise. For this reason, using single input (voltage) to derive equivalents has also been tested. However, the results suggested that both factors have effects on the state of the system.

C. Disturbances

The disturbances used to derive equivalents should be practical (e.g. line fault or tripping, load rejection etc.) and disturbances like bus faults are not suitable for this reason. The disturbances could be grouped into three categories depending on their locations: a) disturbances in the study system; b) disturbances in the external system; c) and disturbances at the connecting bus.

The object system to be replaced by equivalents is the external system. In real life, the external system cannot be disconnected from the study system for testing and measuring purpose. Hence the measurements obtained at the connecting bus actually include the responses of not only the external system but also the study system.

A key point is to find out disturbances, which have minor effects on the study system so that the dynamic responses of the system are mainly the responses of the external system. If we choose disturbances in the study system, the generator oscillations in the study system cannot be neglected. The models that we derive using these measurements are the equivalents of the full system (the study system and the external system) rather than the external system only. Some disturbances in the external system may have small effects on the study system. But the disadvantage of adding disturbances in the external system is that it could change the topology of the object system during the fault time. This will more or less affect the accuracy of the equivalents. Also, these equivalents are highly dependent on the locations of disturbances. The third option is to add disturbances at the connecting bus between the study system and the external system.

Among these disturbances that could be added at the connecting bus, changing loads may have relatively small effects on the study system. Disconnection and reconnection of loads could keep the post-fault steady state the same as the initial steady state. On the other hand, the sole load disconnection would change the steady state operating point. In this case, the dynamic response of the network is the combination of the response to the load change and a step response. Besides, from the practical point of view, load disconnection and reconnection could be tested in the real power system without affecting the network operation very much, if the disconnection time is short enough. Therefore the load disconnection and reconnection disturbances are of the main interests for the dynamic equivalencing method presented in this paper.

In this research we have concentrated on disconnection and reconnection of a reactive load. Real load and complex load disconnection have also been tested as disturbances but disconnecting reactive loads is more practical in real power systems since it could be done by just disconnecting a capacitor bank. Disconnecting a real load means disconnecting a customer which is unacceptable. Additionally, the real component of loads may have some effects on frequency. If the frequency has relatively small effects on the accuracy of the equivalent, the effects of real power on frequency could be neglected.

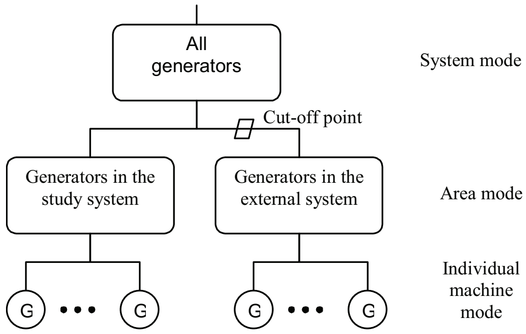

Fig.2. Modal tree structure of a power system

Disturbances like three phases short-circuit fault, tripping branch and closing branch etc. have also been simulated in either the study system or the external system for testing.

D. Modal Analysis

A disturbance which leads to electromechanical oscillations (of generators can be analysed using modal (eigenvalue) analysis. The modes could be illustrated in a tree structure (Fig. 2):

1) System modes: The root modes of the tree are system modes. System modes are non-oscillatory (real) modes. They are the most basic modes, which involve all the generators in the system. The close-to-zero system modes correspond to the angle state. The real-negative system modes correspond to the speed state (system frequency).

2) Inter-area modes: Inter-area mode oscillations are associated with generators in one area swinging coherently against generators in another area. These modes have lower frequencies and are within the range of 0.05 to1Hz.

3) Local modes: Local modes, or machine modes, oscillations are associated with single generators. Their frequencies are typically in a range of 0.5 to 2.5Hz.

The areas could be divided into sub-areas till individual generators are obtained. The modal tree structure shown in Fig. 2 is simplified for clearer illustration.

In order to understand better how the derived equivalent worked, we have undertaken eigenvalue analysis of the full and the reduced power system models. For dynamic equivalencing, any modes that involve any machine in the study system should be kept and all the other modes can be discarded and replaced by the equivalent. That means we could cut off the branch of the external system modes at the point shown in Fig. 2. The retained modes should be the system modes, the area modes and study system machine modes. In other words, the external system machine modes can be discarded.

IV. CASE STUDY

The test system used for this project was a part of the Scottish power system shown in Fig. 3. The study system is the

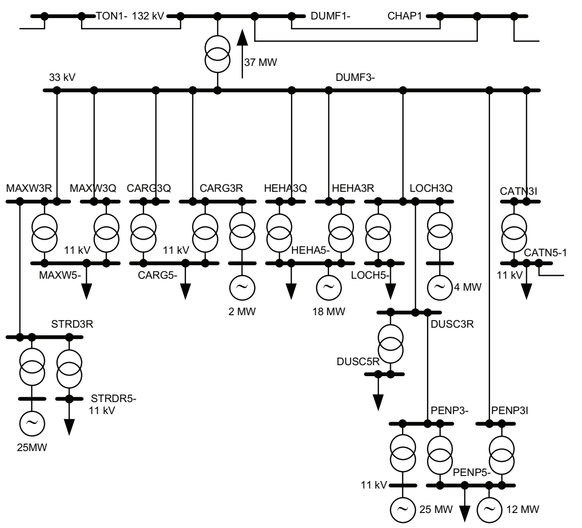

Fig. 3. Distribution power system connected to Dumfries 132 kV (DUMF1-) substation.

transmission network (partly shown at the top pf the diagram)

TABLE I
UNITS IN DISTRIBUTION SYSTEM

Bus	Unit type	Sn[MVA]	Pg[MW]
STRD3R	Steam	30	25
PENP3-	Steam	30	25
CARG3R	Diesel	2.5	2
LOCH3Q	Diesel	5	4
HEHA5-	Hydro	20	18
PENP5-	Hydro	14	12

while the external system is the distribution network shown below the 132/33 kV transformer with generating units listed in Table 1.

After applying a disturbance consisting of disconnecting a 2 Mvar reactive load at the connecting bus, oscillations in both transmission and distribution networks were observed. However the amplitude of real power oscillations in transmission network was in the range of 10^{-5} p.u., which means the oscillations of the generators were really small, just a noise. However, in the distribution network the generators have relatively large oscillations, in the range of 10^{-3} p.u. Hence we concluded that such disturbances, which have really small effects on transmission network, could be used to derive a dynamic equivalent that represents only (or rather mainly) the distribution network.

The whole system was modelled in PSS/E program. Disturbances were introduced into either distribution network or connecting bus. Time series of real power, reactive power, voltage and frequency were measured at the connecting bus. These time series were then imported into Matlab System Identification Toolbox to derive equivalent models, which were later introduced into PSS/E to replace the distribution network for verification purpose. We will refer to two models:

(i) The original model (T+D):

Transmission network is connected with the distribution network. All the generators in the system are represented by the sixth or fifth sub-transient models. Loads are composed of constant real and reactive power.

(ii) The equivalent model (T+Eq):

All the generators in transmission network are represented by the sixth or fifth sub-transient model. Loads are composed of constant real and reactive power. The distribution network is represented by the derived equivalent (in the state-space form).

$$\begin{aligned} x(t+1) &= Ax(t)+Bu(t)+Ke(t) \\ y(t) &= Cx(t)+Du(t)+e(t) \end{aligned} \tag{1}$$

State-space model relates the current output $y(t)$ and the next state $x(t+1)$ to current state variables $x(t)$ and input $u(t)$. $e(t)$ is a white noise.

V. RESULTS AND DISCUSSION

Fig.5 shows the eigenvalues of the power system on s-plane. The squares represent the modes of the whole (T+D) power system. The diamonds represent the modes of the transmission system only which were obtained by representing the whole distribution network by a constant load. By comparing those

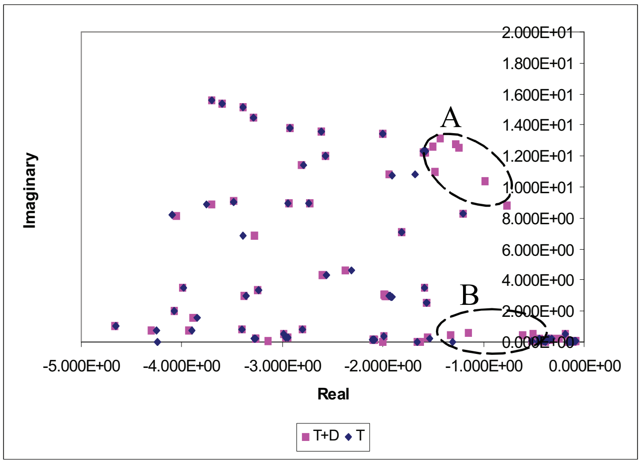

Fig. 5. Mode positions: (T+D) transmission system and distribution system; (T) transmission system only.

modes in the plot we could find the modes associated with distribution system - the diamonds that are not covered by the transmission system modes.

The oscillation modes of the distribution system could be grouped into two groups: a) Group B is close to the real axis and has frequencies lower than 0.5Hz, which suggests that they are inter-area modes; b) Group A has frequencies higher than 1Hz, which is within the typical local mode frequency range.

The modes transferred to the z-plane using sampling time as 0.05s (the time step of discrete model) are shown in the left plot of Fig.6. The crosses and the circles represent the modes of the whole system and the transmission system respectively. We can see that the distribution system modes are distributed at real axis, close to unit circle 0^{o}, and around 30^{o}. As we can see from the polar plot, local modes of the system are around 30^{o}. The distribution system local modes are mixed with transmission system local modes. Hence, it is hard to extract the distribution modes.

The right plot in Fig. 6 shows the modes of the dynamic equivalent of the distribution network identified by the 5th order model (called n4s5) and the 6th order model (called n4s6) which were derived using the same data set. The 5th order model n4s5 has identified the poles at the position corresponding to the system modes and area modes only. The 6th order model n4s6 has also identified a pair of complex poles, which corresponded to the local machines.

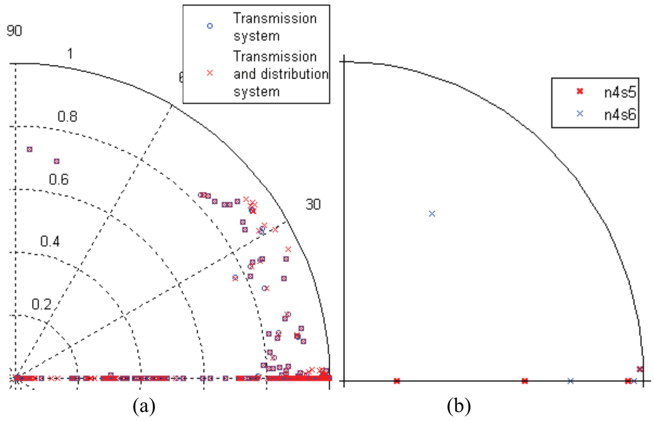

Fig. 6. Z-plane eigenvalues: (a) the whole transmission and distribution system, (b) equivalent only.

As discussed earlier, since we cannot disconnect distribution system from the transmission system, the data sets of system response we use to derive the model is a combined response of both the transmission system and the distribution system. Although we could choose a proper disturbance (close to distribution network) to excite the response of the distribution system more, the response of the transmission network cannot be totally ignored, especially the response of generators near the distribution system.

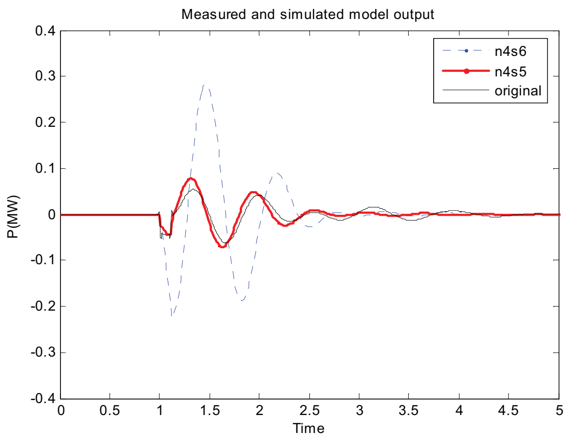

Fig. 7. Comparison of the outputs of models with the 6th order equivalent n4s6 and the 5th order equivalent n4s5

The two equivalent models were implemented into the power system model to replace the original distribution system with their outputs as shown in Fig.7. Clearly the response of the new system with n4s5 fits the response of original system well. On the contrary, n4s6 significantly increases the oscillation amplitude. The likely reason is that n4s6 model contained the modes of local machines, which also included generators in the transmission system. This could have doubled the response of those generators in transmission system when the model is connected with the transmission system, and cause larger oscillations in the system.

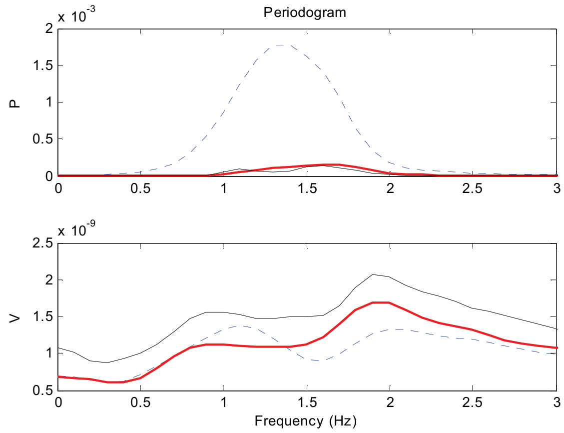

Fig.8. Power spectral density of time series. Solid line corresponds to the original system, dashed line corresponds to n4s5, dotted line corresponds to n4s6.

Fig.8 shows the system response spectra obtained from the original system (solid line), system with n4s5 equivalent (dashed thick line) and system with n4s6 equivalent (dotted line). The upper plot shows real power P flowing from distribution side to bus TON1. The lower plot shows the voltage at bus TON1.

The plots in Fig.8 confirm that model n4s5 seems to offer a better performance than n4s6. The power plots (upper diagram) show a good agreement between the original system and n4s5 while the plot of n4s6 is distinctly different. The voltage plots show that n4s6 model keeps the peak value at around 1Hz rather than around 2 Hz as in the original system. The plot of n4s5 is again much closer to that of the original system. on transmission network. Model n4s5 has a different initial value than the

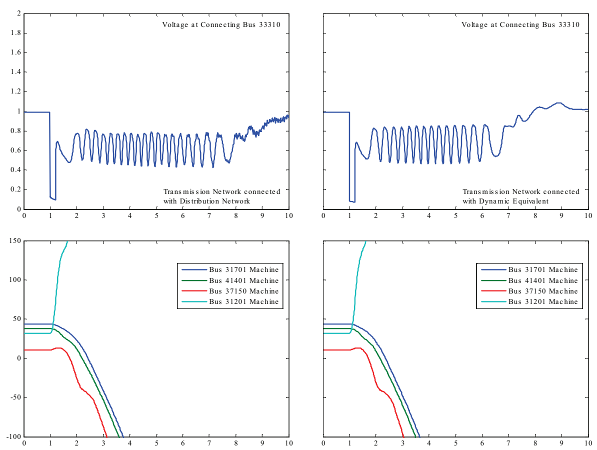

Fig.9 Comparison of original system model (left) and model with n4s5 equivalent (right) in voltage (up) and rotor angles of units in transmission system (down).

original model probably due to different mean values caused by output errors.

Fig.9 compares the response of the original (T+D) model with (T+eq) model when a three phase line fault was applied in the transmission network near the distribution network. The original and the equivalent models have the same maximum clearing time for this fault and the result shows a close resemblance of their voltage and rotor angles transients.

The study that concerns the angle stability of generators in distribution networks has also been carried out. The results suggested that disturbances within the distribution network are required to derive the equivalent for such study.

VI. CONCLUSIONS

Increasing penetration of generators embedded in distribution networks makes it necessary to represent them by dynamic equivalents in power system stability studies. As often the exact model of the distribution network with embedded generators is not known, the aim of this project was to produce a measurement-based, rather than a model-based, dynamic equivalent of a distribution network with embedded generation.

A dynamic equivalencing approach using system identification has been presented in this paper. Following a disturbance consisting of disconnecting a capacitor in the transmission network, a dynamic equivalent has been created. Eigenvalue analysis and time-domain simulations have confirmed a good performance of the equivalent.

VII. APPENDIX

Subspace methods to estimate state-space models are explained in detail in [5].

The state space model structure is

$$\begin{aligned} x(t+1) &= Ax(t)+Bu(t)+w(t) \\ y(t) &= Cx(t)+Du(t)+v(t) \end{aligned} \qquad \text{(A1)}$$

w(t) and v(t) are assumed to be white, have zero mean and a covariance matrix of the form

$$E\left(\begin{bmatrix} w \\ v \end{bmatrix} \begin{bmatrix} w^T & v^T \end{bmatrix}\right) = \begin{bmatrix} Q & S \\ S^T & R \end{bmatrix} \delta_{pq} \qquad \text{(A2)}$$

Based on a finite set of collected data{ut, yt},t=0,…, N-1, N4sid algorithms are concerned to find an estimate for the model order *n* of the system, estimates for the matrices A, B, C, D to a similar transformation , and the noise covariance matrices Q, S and R.

Block Hankel matrices play an important role in these algorithms. The input data are arranged in Hankel matrix as

$$\begin{aligned} U &= \begin{bmatrix} u_0 & u_1 & \cdots & u_j-1 \\ u_1 & u_2 & & u_j \\ \vdots & & \ddots & \\ u_{2i-1} & \cdots & & u_{j+2i-2} \end{bmatrix} \\ &= \begin{bmatrix} U_1 \\ U_2 \end{bmatrix} \end{aligned} \qquad \text{(A3)}$$

This matrix is divided into two halves, which represent the past U1 and the future U2 with respect to a reference t=i

The output Hankel matrix Y, the past input/output block Hankel matrix and the extended observability matrix of the order i are defined as:

$$Y = \begin{bmatrix} Y_1 \\ Y_2 \end{bmatrix} \qquad W_p = \begin{bmatrix} U_1 \\ Y_1 \end{bmatrix} \qquad \Gamma_i = \begin{bmatrix} C \\ CA \\ \vdots \\ CA^{i-1} \end{bmatrix} \qquad \text{(A4)}$$

The state sequence can be obtained from $O_i = \Gamma_i \tilde{X}_i$, where Oi is the oblique projection of Y2 onto Wp along the U2:

$$O_i = Y_2 / U_2 W_p \qquad \text{(A5)}$$

The steps of the algorithm are as follows:

Calculate the oblique projections of the future outputs along the future inputs onto the past. This can be implemented using the least squares algorithm.

Calculate the Singular Value Decomposition (SVD) of the oblique projection Oi and determine the order.

Determine the extended observability matrix.

Determine estimates for the state sequences.

Derive estimates for A, B, C and D.

Determine Q, S and R.

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IX. BIOGRAPHIES

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